

A Market Design Solution to the Unequal Distribution of Teachers in Schools

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Abstract

In most countries, public schools in disadvantaged districts have relatively fewer experienced teachers than those in more privileged districts. As teacher experience is an important indicator of good education outcomes, this presents itself as an important shortcoming of public education. Moreover, many of such countries use centralized matching mechanisms for assigning new teachers to their first jobs at schools and reassigning tenured teachers who would like to move. We address the unfair teacher distribution problem through a market design approach by introducing two new centralized (re)assignment mechanisms. The defining property of these mechanisms is that the final allocation improves not only teachers' welfare but also the distribution of teachers' experience in schools with respect the status quo. While both mechanisms are strategy-proof for teachers, one achieves two-sided Pareto efficiency and in particular teacher optimality and the other one achieves an appropriately defined stability property, targeted for countries that already use stability-based assignment schemes. We empirically estimate teacher preferences using data from the existing assignment system in France and test our proposals' performance using several empirical metrics. We observe that imposing that the distribution of teachers' experience improves with respect the status quo together with a stability property can backfire: these constraints are so demanding that mobility of tenured teacher is almost zero. As a consequence, the experience gap between unattractive and attractive regions may be higher compared to what is achieved by mechanisms which do not impose such constraints. This issue does not occur under our efficient mechanism which successfully reduces the experience gap among attractive and unattractive regions.

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1 Introduction

Most education systems around the world suffer from a common problem: good teachers are not equally distributed between schools. In the United States, good teachers tend to work in schools that serve more affluent students (Hanushek, Kain and Rivkin, 2004, Jackson, 2009). In Norway, England, and France, good teachers prefer schools with a higher share of native and high-achieving students (Bonesrønning, Falch and Strøm, 2005, Allen, Burgess and Mayo, 2018). Some countries try to solve this unequal distribution by making disadvantaged schools more attractive through higher salaries, lower class size, or better working conditions (Biasi, forthcoming, Falch, 2010, OECD, 2005). Yet, the scope to solve teachers’ unequal distribution through higher salaries is often limited by a fundamental constraint: most teachers are civil servants, which implies that their salary is regulated by a fixed pay scale that prevents policy makers from using it as a compensating factor. In such contexts, countries that use centralized matching mechanisms benefit from an additional tool to mitigate this unequal distribution: the centralized assignment system.

In this paper, we design matching mechanisms whose objective is to improve the distribution of teachers in schools (compared to an initial assignment), and we empirically quantify the gains that these mechanisms would bring in a real-life teacher assignment setting. We start by introducing a two-sided matching framework in which we explicitly model schools’ preferences as a proxy for the central authority’s distributional objective. This objective could be to better balance inexperienced teachers across schools for instance. To help define a distribution, we introduce a new concept: Each teacher has a type which captures her observable characteristics that the central authority might want to better balance such as her experience, education, past performance, etc. In addition, in our model, teachers can either be tenured or new. Tenured teachers are initially assigned, which is captured by a *status-quo matching*. New teachers can be new graduates or the teachers who are employed by private schools. As in the standard setting, teachers have preferences over schools.

To incorporate distributional constraints in the model, a novelty of our approach is to consider that schools are part of a resource pool that is collectively managed by a central authority, whose objective is to improve the distribution of teachers in the school system. Schools have “preferences” over sets of teachers, and we think of these preferences as reflecting the central authority’s objective. In many countries, this objective might be to assign experienced teachers to disadvantaged schools where inexperienced teachers are overrepresented and new graduates to schools where experienced teachers are overrepresented in order to improve the experience of new graduates. In that case, in our model, the disadvantaged schools would have a preference ordering over types that ranks teachers by decreasing levels of experience, i.e., the most experienced teachers would always be preferred to the least experienced teachers. Beyond this specific example, each school has a customized preference ordering over teachers’ types. We assume that a school gets better-off if the distribution of types increases according to first-order stochastic dominance. In other words, given a set of teachers assigned to a school and given a schools’ preference over teachers’ types, a school gets better-off if, for each teacher’s type, the fraction of teachers with this type or a more-preferred type increases after the match. By construction, when schools get better off compared to the status-quo

assignment, the distribution of teachers improves. We allow schools to have empty positions in the status-quo assignment.

We impose that our mechanisms produce assignments that improve upon the status-quo for teachers but also for schools: Both sides should get weakly better-off compared to the initial assignment. This defines the class of mechanisms that are *status-quo improving* (SI hereafter). This is an important criterion that ensures that a better welfare of teachers is not achieved at the cost of a poorer distribution of teachers, or vice versa. Status-quo improvement and strategy-proofness (for teachers) are the two fundamental properties we require in our mechanisms. We then build on this by considering two additional criteria that matchings should respect: efficiency (where welfare entities are teachers) and stability. Since these criteria are in conflict (Balinski and Sönmez, 1999), we introduce two mechanisms that are status-quo improving and strategy-proof. One will produce a matching that is efficient for teacher among SI mechanisms; the other one achieves a stable matching where stability is appropriately defined for our environment.

The first mechanism we propose is two-sided Pareto efficient, and in particular, SI teacher optimal, i.e. efficient for teachers among status-quo improving mechanisms. This mechanism, named the status-quo improving cycles and chains (hereafter, SI-CC), is related to top-trading cycles (TTC) mechanisms (in particular, inspired by Gale's TTC of Shapley and Scarf (1974), YRMH-IGYT of Abdulkadiroğlu and Sönmez (1999), and TTCC of Roth, Sönmez and Ünver (2004)) but has one key feature different: While these TTC-type mechanisms ensure that only teachers necessarily get better off as we execute exchanges, both teachers and schools get better off with respect to the status-quo under SI-CC. To reach this objective, we introduce two main innovations in the mechanism. First, we define the schools' pointing rule where the order in which the school would like to send out its initial (aka status-quo) employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned to a different school. We define the pointing order such that less-preferred-type employees are pointed first (therefore leaving first) and more-preferred-type employees are pointed later. The second innovation pertains to the teacher pointing rule, which lets teachers point to schools, and therefore determines which teachers can be assigned to a school. We only allow a teacher to point to a school if replacing the pointed teacher by that school with her does not make the school worse-off.

Next, we turn our attention to stability, noting that stability and status-quo improvement may in general be in conflict (e.g., Compte and Jehiel (2008), Pereyra (2013)). Indeed, due to the individual rationality constraint for teachers, a tenured teacher has the right to stay at her status-quo assignment if she does not obtain any of the schools she ranks. If this tenured teacher is disliked by all schools (including her initial school), blocking pairs may form. To overcome the conflict between stability and individual rationality, the standard approach consists in weakening stability by ignoring blocking pairs that where assigning the teacher to the school of the blocking pair would displace a status-quo employee. We show that when imposing status-quo improvement in contexts where schools may initially have vacant seats, this weakening alone does not resolve the conflict. We introduce a stability notion which implicitly gives new teachers rights over status-quo empty seats of a school. Under a mild overdemand assumption involving new teachers and schools with

excess status-quo capacity, we show the existence of a strategy-proof and SI-stable mechanism, the status-quo improving deferred acceptance mechanism (hereafter, SI-DA).¹ Although it may appear counter intuitive to give a new teacher priority for the empty seats of a school, we show by means of examples that this is necessary to sustain status-quo improvement. Moreover, the main reason for giving priority to new teachers for the empty seats is achieving status-quo improvement for the schools.

To define the SI-DA mechanism, we need to define auxiliary choice functions for schools. To this end, we first distribute status-quo employees to individual positions of the school, and consider an order of precedence among these positions based on the desirability of the teacher occupying the position. Vacant status-quo positions of a school, if there are any, are placed at the very end of this order of precedence. We construct an auxiliary “preference” ranking for each position such that some teachers are unacceptable and each status-quo teacher is ranked first by her position. The rest of the preference ranking for each occupied position is determined by making only the teachers who are at least as good as the occupying teacher acceptable and ranked according to desirability just below this teacher. For the vacant positions, new teachers are ranked first in terms of their desirability and then the remaining teachers. When a set of applicants apply a school in the SI-DA mechanism, its auxiliary choice function fills positions according to the precedence order of positions: the first position gets the most desirable acceptable applicant in terms of its auxiliary preference ranking, the second position gets the most desirable acceptable applicant among the rest in terms of its auxiliary preference ranking, and so on. The construction of this auxiliary choice function has similarities with the use of slot specific priorities model introduced by Kominers and Sönmez (2016).

Applicability of our theoretical framework is not only limited to centralized teacher assignment. In fact, our framework can be applied to any centralized two sided matching market in which status-quo assignment is aimed to be improved for both sides of the market. Examples of such markets are, including, but not limited to, student exchange programs between colleges, public school districts targeting racial balances among schools, and rotational task allocations to employees. Moreover, our model does not restrict the way schools value the experience of the teachers. Hence, our results hold as long as each school has rankings based on a coarse metric of characteristics of teachers such that different schools use possibly different metrics.

In the second part of the paper, we quantify the gains that our mechanisms bring by using data on the annual assignment of teacher to regions in France. Like many other countries, France uses a centralized process to assign teachers to regions and then to schools.² This labor market is particularly appropriate to study our question because it suffers from severe imbalance in the distribution of teachers. About 50% of the tenured teachers who ask to change region come from

¹The overdemand assumption and giving higher priority to new teachers over status-quo empty seats restrict currently employee teachers to flee away their status-quo assignments without being replaced. In the absence of such a restriction some schools might be worse off compared to their status-quo assignment.

²Countries that use a centralized process to assign teachers to schools include Germany, Czech Republic (Ceclárová, Fleiner, Manlove, McBride and Potpinková, 2015), Italy (Barbieri, Rossetti and Sestito, 2011), Turkey (Dur and Kesten, 2014), Mexico (Pereyra, 2013), Peru, Uruguay (Vegas, Urquiola and Cerdán-Infantes, 2006), and Portugal.

two regions (out of 25)—called Creteil and Versailles—that are particularly disadvantaged and therefore unattractive. As a result of this imbalance in exiting request, every year, a majority of the new teachers are assigned one of these two regions to compensate for the large exit flows. This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France and it is seen as one of the structural determinants of the large achievement inequalities that France suffers from.³ Reducing the unequal distribution of teachers across regions became one of the objectives of the French policy makers, who see this as a way to both reduce achievement inequalities and to improve the attractiveness of the teaching profession in the longer-run.

We start the empirical analysis by structurally estimating teachers’ preferences over the French regions. A number of papers show that assuming that teachers truthfully report their preferences is a strong assumption, even when the mechanism is strategy-proof (as in France).⁴ To avoid the potential bias generated by teachers untruthful reports, we estimate teachers preferences under a weaker “stability assumption” developed by Fack, Grenet and He (2019) and applied to the teacher labor market by Combe, Tercieux and Terrier (2020). We estimate the preferences separately for 5,833 teachers who have an initial assignment—referred to as “tenured teachers”— and 4,627 teachers do not have an initial assignment—referred to as “new teachers”. The estimations reveal interesting differences in the preferences of these two groups of teachers. While tenured teachers strongly dislike the Creteil and Versailles regions, these regions are more attractive for new teachers, who might see a first position in a disadvantaged school as a stepping stone for better positions in the future. This difference in preferences surely contributes to the unequal distribution of teachers denounced by policy makers. The counterfactual analysis shows that, above and beyond these preferences, the mechanism used also shapes the distribution of teachers in important ways.

We use the estimated preferences, along with data on regions priorities and vacant positions, to run the two algorithms we propose: SI-CC and SI-DA. We define a teacher type as her experience and assume that regions with relatively young teachers’ body have a preference ordering over types that ranks teachers by decreasing levels of experience, i.e., the most experienced teachers would always be preferred to the least experienced teachers. For regions with a relatively old teachers’ body, we assume on the contrary that they rank teachers by increasing levels of experience. Recall that SI-CC and SI-DA are both status-quo improving, i.e., they impose that the distribution of teachers’ experience must improve upon the initial distribution. One important goal behind this constraint is to produce a more equal distribution of teachers across regions. Of course, imposing status-quo improvement may have a cost in terms of teachers welfare (for instance, measured by the mobility of tenured teachers or the distribution of ranks in their preferences of the regions teachers obtain). To measure the effect of imposing status-quo improvement on both the distribution of teachers’ experience as well as on teachers welfare, we run two benchmark mechanisms, TTC* and

³The PISA results show that, in OECD countries, a more socio-economically advantaged student scores 39 points higher in math than a less-advantaged student, which is equivalent to one year of schooling. There is a large variation between countries in how much a student social background predicts her achievement, and France is one of the worst countries on this inequality indicator, ranking in fourth position (starting from the bottom).

⁴The French ministry of education uses a modified version of the deferred acceptance mechanism.

DA*, which correspond to SI-CC and SI-DA when we do not impose status-quo improvement for schools (we only keep the status-quo improvement for teachers). As explained below, our results vastly differ when considering SI-CC or SI-DA.

When focusing on SI-CC we observe that, imposing status-quo improvement, improves the distribution of teachers' type and, relatedly, reduces the experience gap between unattractive and attractive regions. Further, we show that mobility reduces when imposing SI. More specifically, starting with unattractive regions, SI-CC assigns less inexperienced teachers to these regions than its benchmark which does not require SI. To illustrate the magnitudes, SI-CC only assigns 1,371 teachers with one or two years of experience to the three youngest regions, while TTC* assigns 1,844 of them to these three regions. We find a similar pattern for attractive regions, i.e., SI-CC assigns less experienced teachers to these regions compared to its benchmark TTC*. Finally, SI-CC reduces the gap in teachers experience between young disadvantaged regions and older regions compared to its benchmark mechanism which does not impose SI. We then investigate whether achieving a better distribution is done at the cost of a lower welfare for teachers, as measured by their mobility and the rank of the region they obtain. In line with the existence of a distribution-efficiency trade-off, fewer tenured teachers manage to move under SI-CC than under the benchmark TTC*, but the difference is somewhat limited (1,598 versus 2,470 teachers). The distribution of ranks that tenured teachers obtain under the benchmark also stochastically dominates the one under SI-CC. Interestingly, the opposite is true for new teachers who obtain better ranks under SI-CC.⁵ To conclude, while we are able to quantify the trade-off between teachers' mobility and the distribution of teachers' experience empirically, qualitatively, the trade-off observed is conform to our expectations.

The picture is radically different for SI-DA. Indeed, we believe that one of the most interesting insight of our empirical exercise is the observation that imposing SI to DA-based mechanisms can backfire. More specifically, we first show that imposing SI has a tremendous mobility cost on SI-DA: only 7 tenured teachers move from their initial position under SI-DA, compared to 1,267 under its benchmark DA*. In addition, in the youngest three regions, SI-DA produces a distribution of teachers' experience which does not dominate the distribution of DA*. In the three oldest regions, our results are even more striking: DA* produces a distribution of teachers' experience which dominates the distribution under SI-DA. Put differently, under our DA-based mechanisms, status-quo improvement fails to achieve its goal: the distribution of teachers does not improve and the experience gap between unattractive and attractive regions does not reduce. To understand the phenomenon, recall our previous observation that imposing both SI and stability reduces mobility of tenured teachers almost to zero. In addition, without status-quo improvement, we show that the mechanism only mildly violates status-quo improvement while significantly increasing mobility. These additional mobility gains yield better distribution of teachers in many regions which helps in reducing inequalities across regions.

Related literature. SI-CC mechanism has its roots in the top-trading cycles algorithms (see

⁵We explain this by the fact that, due to larger mobility under the benchmark, tenured teachers are more likely to leave the unattractive regions of Creteil, which mechanically increases the need to assign new teachers to these regions. This lowers the rank of the region obtained because unattractive regions are often ranked relatively low by teachers.

Shapley and Scarf, 1974, Abdulkadiroğlu and Sönmez, 1999, Roth et al., 2004, Dur and Ünver, 2019) and SI-DA mechanism has its roots in the teacher proposing deferred acceptance algorithm (see Gale and Shapley, 1962, Abdulkadiroğlu and Sönmez, 2003a, Kominers and Sönmez, 2016).

Our stability-based approach using SI-DA is related to the literature on stable allocation under distributional constraints. In the school choice literature, Abdulkadiroğlu and Sönmez (2003b) introduce assignment schemes imposing type-specific ceilings at schools. Other related papers are Abdulkadiroğlu (2005), Kojima (2012), Hafalir, Yenmez and Yildirim (2011), Ehlers, Hafalir, Yenmez and Yildirim (2014), Kamada and Kojima (2015), Dur, Kominers, Pathak and Sönmez (2018), Sönmez and Yenmez (2019), and Dur, Pathak and Sönmez (2020). While the focus in these papers is on assignment schemes to achieve diversity and other distributional objectives mostly in school choice and government-mandated job allocation context, our work applies to a teacher assignment problem where there is an initial matching and concern of making both sides better off. This makes our model and analysis different from the existing ones. Our methodology for constructing the SI-DA mechanism and the choice function is inspired by the choice function constructions in slot-specific priorities model of Kominers and Sönmez (2016), which is also used in the latter three aforementioned papers. In these papers, the choice functions and its inputs are preliminaries of the problem. However, in our framework, neither school choice functions nor the inputs are given to us. In particular, our choice function is defined in this way to satisfy desired properties under SI-DA mechanism.

The design of efficient mechanisms in two-sided matching markets with a possible status-quo matching constraint was previously studied by Dur and Ünver (2019) in the context of student and worker exchange programs. The main difference between that model and the current model is that status-quo improvement was not a constraint in this previous paper. This substantially changes the modeling choices and mechanism design. Moreover, we have school preferences based on the first-order stochastic dominance relation over distributions of teacher types leading to a new class of pointing rules. Our design of efficient mechanisms in this domain gives in general higher welfare for the schools. We additionally focus on a stability-based approach with SI-DA in addition to efficient mechanism design and conduct a thorough empirical analysis.

It is through our notion of improvement with respect to status-quo that we achieve a better distribution of teachers in the school system. A related approach is followed in Combe, Tercieux and Terrier (2020) where a teacher assignment problem is also studied. In this paper, they introduce a class of TTC like mechanism, the Teacher Optimal Block Exchange (TO-BE) mechanisms. Their main focus is on two-sided efficiency but they show that a unique selection in this class of TO-BE mechanisms is teacher optimal. They show that it outperforms—in terms of distribution of teachers as well as in terms of efficiency—the assignment scheme used in France which is a variation on the DA mechanism. First, this result is shown by focusing mainly on teachers having an initial assignment, therefore, largely ignoring the imbalance issues that new teachers can create in terms of distribution of teachers. The generalization they propose to account for new teachers is a combination of TO-BE and the current French mechanism based on DA, which is further away from our SI-CC proposal. In addition, the preferences of schools we consider, based on the first-order stochastic dominance

relation, can be seen as a generalization of the one they introduce and the two indeed coincide in their framework. Interestingly, even in their framework, we can show that our SI-CC mechanism is not in the class of TO-BE mechanisms and so can be viewed as another strategy-proof and teacher optimal mechanism (see Example 6 in Appendix B). Second, we define SI-DA which requires improvement with respect to the status-quo matching that the current algorithm used in France does not impose. This mechanism can be viewed as the right benchmark to which one should compare SI-CC in our framework.

Despite these two previous studies and the current paper, the study of efficient mechanisms under distributional constraints is still rare. Suzuki, Tamura, Hamada and Yokoo (2017) and its generalization by Hafalir, Kojima and Yenmez (2017) provide sufficient conditions on policy goals to get a version of TTC that take constraints into account and satisfies desirable properties. In particular, these sufficient conditions involve a notion of discrete convexity on the policy goals, namely, M-convexity. In our context with new teachers and vacant positions at schools, we show that M-convexity of the policy goals is not sufficient anymore to ensure a well-behaved version of TTC (see Example 7 in Appendix B).

Finally, our paper builds on a recent literature developing demand estimation methods in school choice environments (Abdulkadiroğlu, Agarwal and Pathak (2017); Agarwal and Somaini (2018); Calsamiglia, Fu and Güell (2020)). In particular, we build on techniques based on discrete choice models with personalized choice sets which are relevant for preference estimation when reported preferences might fail to be truthfull even under strategy-proof mechanisms (Fack, Grenet and He (2019); Akyol and Krishna (2017); Artemov, Che and He (2019)).

2 Model

2.1 A Teacher Reassignment Market

Let T be a finite **set of teachers**. Each teacher t has a **type**. The type of a teacher captures her all observable characteristics that matter for the schools, such as experience, education, past performance, etc, or only a subset of these.⁶ Let $\Theta = (\theta_1, \theta_2, \dots, \theta_n)$ be the finite type space. Let $\tau : T \rightarrow \Theta$ be the **type function** and $\tau(t)$ be the type of teacher t . For any $\hat{T} \subseteq T$, we denote type θ teachers in \hat{T} with \hat{T}^θ , i.e.,

$$\hat{T}^\theta \equiv \{t \in \hat{T} : \tau(t) = \theta\}.$$

Let S be a finite **set of schools**. Each school s has a **capacity** of q_s . Let $q = (q_s)_{s \in S}$. Each teacher t has a **strict preference relation**, which is a linear order and denoted by P_t , over the schools and **being unassigned option** denoted by \emptyset . Let $P = (P_t)_{t \in T}$. We denote the at least as good as relation related with P_t by R_t for all $t \in T$: $s R_t s'$ if and only if $s = s'$ or $s P_t s'$.

A **matching** $\mu : T \rightarrow S \cup \{\emptyset\}$ is a function such that $|\mu^{-1}(s)| \leq q_s$. With a slight abuse of notation, we use μ_t and μ_s instead of $\mu(t)$ and $\mu^{-1}(s)$, respectively.⁷ We refer to μ_t as the **match**

⁶For example, in the French application, the experience level of a teacher can be thought as the type of a teacher.

⁷Thus, μ_s^θ is the set of teachers of type θ that are assigned school s .

of teacher t and μ_s as the **match** of school s in matching μ . Also for a subset of teachers \hat{T} , we denote the set of their matches in μ , $\mu(\hat{T})$ as $\mu_{\hat{T}}$.

Initially some teachers are already employed by some schools. This is captured by a **status-quo matching** ω . If $\omega_t = s$, then teacher t is currently employed at school s . If $\omega_t = \emptyset$, then teacher t is called a **new teacher**. She is seeking employment for the first time and she is unemployed at the status quo. By definition, $|\omega_s| \leq q_s$ for each school s . We denote the set of new teachers with N , i.e.,

$$N \equiv \{t \in T : \omega_t = \emptyset\}.$$

The rest of the teachers are referred to as **status-quo employees**.

We make one assumption on the preferences of teachers: We assume that $\omega_t P_t \emptyset$ for each $t \in T \setminus N$, i.e., each employed teacher at the status quo matching finds her current school acceptable.

Finally, we define the preferences of schools over subsets of teachers. Unlike teacher preferences, these preferences are typically weak and allow indifferences. Typically \succsim_s denotes the preferences of a school s over subsets of teachers. Let \sim_s and \succ_s be the associated indifference and strict preference relation with \succsim_s , respectively, denoting symmetric and asymmetric portions of the preferences.

To this end, each school s has a **type ranking**, which is a linear order and denoted by \triangleright , over the types of teachers and an **individual rationality threshold** type denoted by θ_\emptyset : $\theta \triangleright_s \theta' \triangleright_s \theta_\emptyset$ means school s ranks type θ teachers over type θ' teachers and finds both types of teachers **acceptable** to hire but it considers type θ' teachers **unacceptable** to hire. Let $\theta \underline{\triangleright}_s \theta'$ if either $\theta \triangleright_s \theta'$ or $\theta = \theta'$. We assume that if $\omega_s^\theta \neq \emptyset$, then $\theta \triangleright_s \theta_\emptyset$, i.e., each school finds the types of its current teachers acceptable.

We make two assumptions on the preferences of schools.

1. We assume that when a school compares two subsets of teachers it uses **first-order stochastic dominance (FOSD)**⁸ relation based on its type ranking. In particular, school s weakly prefers subset of teachers \bar{T} to \hat{T} , i.e., $\bar{T} \succsim_s \hat{T}$, if

- (i) there does not exist an unacceptable teacher in \bar{T} , i.e., $\bar{T}^\theta = \emptyset$ for any $\theta \triangleleft_s \theta_\emptyset$, and
- (ii) for any $\theta \triangleright_s \theta_\emptyset$ we have

$$\sum_{\theta' \underline{\triangleright}_s \theta} |\bar{T}^{\theta'}| \geq \sum_{\theta' \underline{\triangleright}_s \theta} |\hat{T}^{\theta'}|.$$

Moreover, the preference is strict if at least one of the inequalities is strict.

If FOSD does not hold between two sets in either direction, then the school preferences do not compare them. Therefore, school preferences are incomplete. Thus, a school only unambiguously prefers groups whenever two groups of teachers can be ranked based on this FOSD comparison.

In the rest of our analysis, we compare outcome matchings with the status-quo matching. FOSD relation will be sufficient and we will achieve unambiguous comparisons for our purposes.

⁸Although FOSD relation is, in general, defined to compare statistical distribution functions, with a slight abuse of terminology we use the same name for the analogous binary relation that compares distributions of teacher types.

2. We assume that for any subset of teachers T' , if $\theta_{\emptyset} \triangleright_s \tau(t)$ for some $t \in T'$, then $\omega_s \succ_s T'$. That is, each school prefers its status-quo match to any teacher set with an unacceptable teacher in it.

We refer to the list $\langle T, S, q, \omega, P, \succ \rangle$ as a **teacher reassignment market**. Typically, T, S, q, ω, \succ are commonly known in our applications. Only teacher preferences are private information. For the rest of our analysis, we fix T, S, q, ω, \succ and denote a market with teacher preferences P .

We are seeking a matching outcome given a market P .

The most basic property of outcome matchings we consider is *status-quo improvement*. Maybe surprisingly, this is sometimes in conflict with many other standard desiderata used in the literature for matching market design. A matching μ is **status-quo improving** if $\mu_t R_t \omega_t$ for all $t \in T$ and $\mu_s \succ_s \omega_s$ for all $s \in S$. That is, each agent should be weakly better off in a status-quo improving matching with respect to the status-quo matching.

We inspect rules that select a matching for each market. Formally, a (**direct**) **mechanism** ϕ is a function that chooses an outcome matching for any market P . Let $\phi(P)$, $\phi_t(P)$, and $\phi_s(P)$ denote the matching selected by mechanism ϕ under market P , the match of teacher t , and the match of school s in that matching, respectively.

A mechanism ϕ is **strategy-proof** if truth-telling is a weakly dominant strategy for all teachers, that is, for all markets P , for all teachers t , for all possible alternative preference reports P'_t ,

$$\phi_t(P_t, P_{-t}) R_t \phi_t(P'_t, P_{-t}).$$

As we assume that schools' rankings over the types of teachers, and therefore, their preferences over the teachers are commonly known, schools do not need to report them.

In the next two sections, we provide two different mechanisms to achieve two different desiderata: a refinement of Pareto efficiency or an appropriate stability concept for our applications together with status-quo improvement, respectively.⁹

3 Efficiency, Pointing Rule Design, and Status-quo Improving Cycles and Chains

3.1 Status-quo Improving Teacher Optimality

Consider a market P . A matching μ **Pareto dominates** a matching ν **for teachers** if

$$\mu_t R_t \nu_t \text{ for all } t \in T, \text{ and} \tag{1}$$

$$\mu_{t'} P_{t'} \nu_{t'} \text{ for some } t' \in T. \tag{2}$$

⁹In Example 8 in Appendix B, we show that there does not exist a mechanism satisfying efficiency and stability.

Matching μ **Pareto dominates** a matching ν **for schools** if

$$\mu_s \succsim_s \nu_s \text{ for all } s \in S, \text{ and} \tag{3}$$

$$\mu_{s'} \succ_{s'} \nu_{s'} \text{ for some } s' \in S. \tag{4}$$

Finally, matching μ **Pareto dominates** a matching ν if (i) Equations 1 and 3 hold, and (ii) Equation 2 or Equation 4 holds. A matching is **(two-sided) Pareto efficient** if it is not Pareto dominated by any other matching.

A matching μ is **status-quo improving – teacher optimal (SI teacher optimal for short)** if it is status-quo improving and not Pareto dominated for teachers by any other status-quo improving matching. While SI teacher optimality seems to care mostly about the welfare of teachers, any SI teacher optimal matching is also Pareto efficient, since teacher preferences are strict.

Proposition 1 *Any SI teacher optimal matching is Pareto efficient.*

All proofs are provided in Appendix A.

We will introduce a strategy-proof and SI teacher optimal mechanism. Why do we implement on SI teacher optimality rather than a dual concept such as SI-school optimality or different Pareto efficient outcomes?¹⁰ It has two reasons: First, in most of the applications we have in mind, the side we characterize as schools are quasi-agents rather than full agents unlike the side we characterize as teachers. Their welfare has been the main efficiency measure both in practice and in literature (for example, see Abdulkadiroğlu and Sönmez, 2003a, Combe, Tercieux and Terrier, 2020). Second, if we wanted to implement other Pareto efficient outcomes that are not SI teacher optimal, we would not be able to find a strategy-proof mechanism.¹¹

3.2 Status-quo Improving Cycles and Chains Mechanism

Next, we will introduce a strategy-proof and SI teacher optimal mechanism. To achieve this goal, we introduce additional tools.

Our mechanism will iteratively construct a sequence of directed graphs in which teachers, schools, and being unassigned option are the nodes. Teachers can only point to schools or the being unassigned option and schools can only point to teachers in their status-quo match in each of these graphs. When node x points to node y , then a directed arc from x to y is activated.

Our mechanism relies on *executing* two types of multi-lateral exchanges based on the constructed directed graphs.

A **cycle** is a directed path of distinct teachers $\{t_m\}$ and distinct schools $\{s_m\}$, possibly being unassigned option \emptyset ,

$$(s_1, t_1, s_2, t_2, \dots, s_k, t_k)$$

¹⁰Because of indifference classes in schools' FOSD preferences regarding same-type teachers, this concept defined as a simple dual notion of SI teacher optimality may not be Pareto efficient and has to be refined even further.

¹¹In Appendix A, Proposition 6 shows that if a status-quo improving mechanism selects a Pareto efficient matching other than SI teacher optimal whenever such a matching exists, then this mechanism can be manipulated by teachers.

such that $\omega_{t_m} = s_m$ for all m , each node points to the next node in the path, and t_k points back to s_1 .

A **chain** is a directed path of distinct teachers $\{t_m\}$ and schools $\{s_m\}$

$$(t_0, s_1, t_1, \dots, s_{k-1}, t_{k-1}, s_k)$$

such that $\omega_{t_m} = s_m$ for all $m = 1, \dots, k-1$, each node points to the next node in the path.¹² Here, we say the chain starts with t_0 and ends with s_k . In other words, s_k is the head of the chain and t_0 is the tail of the chain.

As certain cycles and chains are encountered in the constructed graph, we will **execute** the exchanges in them by assigning each teacher to the school she is pointing to and remove her.

Our main theoretical innovation relies on designing *pointing rules* that designate which possible directed arcs in a graph will be endogenously activated through the algorithm.

Pointing rule of teachers will be introduced within the definition of the mechanism below as it uses the endogenous working of the mechanism's algorithm. On the other hand, the pointing rule of schools relies on their type rankings and an exogenously given tie breaker.

Formally, a **tie breaker** is a linear order \vdash over teachers.¹³ It can be randomly determined or can be the mandated priority orders for a particular application, such as in the French case, or can be exogenously fixed in some other manner.

The tie breaker for the new teachers and tie breaker regarding teachers employed at the status quo are utilized differently in the algorithm. For each school s , using tie breaker \vdash and its type ranking \triangleright_s , we first construct a **pointing order** \triangleright_s over teachers in ω_s , which is a linear order as well: For any two distinct teachers $t, t' \in \omega_s$,

$$t \triangleright_s t' \iff \tau(t) \triangleleft_s \tau(t') \quad \text{or} \quad [\tau(t) = \tau(t') \text{ and } t \vdash t'].$$

Note that a worse-type teacher is prioritized over a better-type teacher, and only when two same-type teachers are compared, we use the tie breaker to prioritize one over the other.

As the mechanism will iteratively assign and remove teachers, the **pointing rule of schools** is “point to the highest remaining priority teacher in its priority order.”

Now, we are ready to define our mechanism through an iterative algorithm:

Definition 1 *Status-quo Improving Cycles and Chains (SI-CC) Mechanism*

We will construct a matching μ dynamically through the following algorithm. Initially, μ is the empty matching, in which no teacher is assigned to any school. In each step, as teachers are assigned in μ , they will be removed from the algorithm; similarly schools whose all seats are filled in μ and also some other schools chosen by the algorithm will be removed.

¹²Notice that, we allow a school to appear more than once in a chain.

¹³Technically, each tie breaker induces a new mechanism in our class.

For each school s and type θ , let b_s^θ track the **current balance** of type θ teachers at school s in **current matching** μ , which is the matching fixed until the beginning of the current step. The current balance is defined as the difference between the number of type θ teachers assigned to s in μ and the number of type θ teachers in its status-quo match assigned to any school in μ : $b_s^\theta \equiv |\mu_s^\theta| - |\{t \in \omega_s : \mu_t \neq \emptyset\}^\theta|$. Thus, we initialize $b_s^\theta = 0$.

A general step k is defined as follows:

Step k :

- Each remaining school s points to the highest priority remaining teacher in ω_s under \succ_s , if not all students in ω_s are already assigned in μ ; let t_s^k be the teacher pointed to by school s in step k . Otherwise, school s does not point to any teacher.
- We define the **pointing rule of teachers** as follows: Any remaining teacher t is allowed to point to a remaining school s if at least one of the following two school **improvement conditions** hold for school s via teacher t :

1. (Improvement for s by teacher trades) if the school points to a teacher t_s^k and

$$\sum_{\theta' \succeq_s \theta} b_s^{\theta'} > 0 \text{ for all types } \theta \text{ such that } \tau(t_s^k) \succeq_s \theta \succ_s \tau(t),$$

or¹⁴

2. (Improvement for s by only incoming teachers) $\tau(t) \succ_s \theta_\emptyset$, school s currently has an unfilled seat, i.e., $q_s - |\mu_s| > |\{t' \in \omega_s : \mu_{t'} = \emptyset\}|$, and there are remaining new teachers.

Let A_t^k be the opportunity set for a remaining teacher t , i.e., the set of schools t can point to in this step together with the being unassigned option \emptyset .¹⁵

Each remaining teacher t points to her most preferred option in A_t^k .

- Being unassigned option \emptyset points to all teachers pointing to it.

Due to finiteness, there exists either

- (i) a cycle in which all schools in the cycle satisfy improvement Condition 1 or a cycle between a single teacher and the being unassigned option \emptyset , or
- (ii) a chain.

Then:

- **If Case (i) holds:** Each teacher can be in at most one cycle as she points at most to a single option. We execute exchanges in each cycle encountered in case (i) by assigning the teachers in that cycle to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools in μ , and go to step $k + 1$.
- **If Case (i) does not hold:** Then case (ii) holds, i.e., there exists a chain. In particular, each remaining teacher initiates a chain. There are two subcases:

¹⁴Condition 1 is trivially satisfied if no such θ exists, i.e., if $\tau(t) \succeq_s \tau(t_s^k)$. An alternative condition to this one is provided in Appendix C.

¹⁵Note that, $\omega_t \in A_t^k$ for all remaining teachers t who were employed at the status quo.

– **If there exists a remaining new teacher:** Then we select a chain to be executed as follows:

- * Select as the tail of the chain the new teacher with the highest priority under tie breaker \vdash and then include in the chain the school she points to.¹⁶ If Improvement Condition 1 does not hold for this school via this teacher, but only Improvement Condition 2 holds, then we end the chain with this school; otherwise, we repeat the following:
- * Include to the chain the teacher pointed by the last school included.¹⁷ If we include a teacher, we also include next in the chain the school she is pointing to. We repeat this iteratively until the Improvement Condition 1 does not hold for the next school via the included teacher, but only Improvement Condition 2 holds.¹⁸

The last school included is the head of the selected chain.

We execute the exchanges in the selected chain by assigning each teacher in the chain to the school she points to, update current matching μ and current balances $\{b_s^\theta\}$ accordingly, remove assigned teachers and filled schools, and go to step $k + 1$.

– **If there does not exist a remaining new teacher:** Then we remove each school s whose all status-quo employees in ω_s were already assigned in μ .¹⁹ We continue with step $k + 1$.

The mechanism terminates when all teachers are removed. Its outcome is the final matching μ .

The name of the mechanism suggests that both teachers and schools become better off through the mechanism with respect to the status quo. Indeed, this is the case. We introduced several innovations in the mechanism that exploit different Pareto improvement possibilities for teachers and schools over the status-quo matching.

Pareto improvement of teachers is straightforward in the algorithm. Teachers who are employed at the status quo are eventually assigned to a school at least as good as their status-quo match. Moreover, all teachers are assigned the best option they can point in the step they are assigned.

What is more delicate is the Pareto improvement of schools, i.e., how we make sure that they always weakly improve with respect to their status-quo match in every step. This is ensured through the introduction of both teacher and school pointing rules.

A school's pointing order designates in which order the school would like to *send out* its status-quo employees. By pointing, the school effectively gives permission to one of its status-quo employees to be assigned possibly to a different school. Thus, we make sure that this priority order is in reverse order of its preferences: Less preferred-type employees are pointed first and more preferred-type employees are pointed later. This is the first innovation.

¹⁶Such a school exists, because if she does not point to a school, then she pointed to being unassigned option \emptyset and was removed previously.

¹⁷Such a teacher exists by Improvement Condition 1.

¹⁸This iterative procedure is guaranteed to terminate. Otherwise, we would have a cycle.

¹⁹There must exist a school which is still in the market and whose status-quo employees have already been assigned. Otherwise, each remaining school would point to one teacher and each remaining teacher would point to a school and so there would exist a cycle, a contradiction.

On the other hand, the teacher pointing rule designates which teachers can be assigned to a school. Therefore, we only allow teachers who can improve the school's welfare with respect to its status-quo match after the currently pointed employee of the school is sent out.

The two school improvement conditions make sure of this.

Condition 1 has two cases: If the type of the possibly incoming teacher is at least as good as the type of the possibly outgoing teacher, the school has no danger of becoming worse off in this trade. The second case on the other hand is more delicate: As trades that strictly improve a school's welfare occur over steps, schools acquire new teachers who are actually of better types than the types of outgoing status-quo employees. Therefore, they may build up a *buffer*. If such a buffer exists, a worse-type teacher than its currently outgoing employee can still be assigned to the school, although this trade makes the school worse off with respect to the previous step. However, the school is still weakly better off with respect to the status quo thanks to the buffer. Only the buffer gets thinner. The existence of the buffer is tracked by checking whether the sums of the relevant type balances, b_s^θ 's, are positive through Condition 1. The use of this buffer ensures teacher optimality.

While the first condition is about a trade the school will make by exchanging an outgoing teacher with an incoming teacher, Condition 2 is only relevant as long as new teachers remain in the algorithm. When Condition 2 holds for a school via some teacher, but not Condition 1, the school will not send out an employee as it has extra capacity: it will only hire one additional acceptable teacher.

Before stating our main result, we illustrate how the SI-CC mechanism works using Example 1.

Example 1 Let $S = \{s_1, s_2, s_3, s_4\}$, $T = \{t_1, t'_1, t_2, t'_2, t_3, t, t'\}$, the status-quo matching be

$$\omega_{s_1} = \{t_1, t'_1\}, \omega_{s_2} = \{t_2, t'_2\} \omega_{s_3} = \{t_3\}, \omega_{s_4} = \emptyset,$$

$q_{s_1} = 3$, $q_{s_2} = q_{s_4} = 2$, and $q_{s_3} = 1$. The preferences of teachers are:

$$s_2 P_{t_1} s_1 P_{t_1} \emptyset P_{t_1} s_3 P_{t_1} s_4$$

$$s_4 P_{t'_1} s_1 P_{t'_1} \emptyset P_{t'_1} s_2 P_{t'_1} s_3$$

$$s_3 P_{t_2} s_2 P_{t_2} \emptyset P_{t_2} s_1 P_{t_2} s_4$$

$$s_1 P_{t'_2} s_2 P_{t'_2} \emptyset P_{t'_2} s_3 P_{t'_2} s_4$$

$$s_4 P_{t_3} s_2 P_{t_3} s_3 P_{t_3} \emptyset P_{t_3} s_4$$

$$s_1 P_t \emptyset P_t s_2 P_t s_3 P_t s_4$$

$$s_1 P_{t'} \emptyset P_{t'} s_2 P_{t'} s_3 P_{t'} s_4$$

There are only two types of teachers: $\bar{\theta}$ and $\underline{\theta}$ for respectively "High" and "Low" experience. All schools prefer high experience teachers to low experience teachers so that $\bar{\theta} \triangleright_{s_i} \underline{\theta}$ for $i = 1, 2, 3, 4$. We assume that $\tau(t'_i) = \tau(t_3) = \underline{\theta}$ for $i = 1, 2$, $\tau(t_i) = \bar{\theta}$ for $i = 1, 2$ and $\tau(t) = \tau(t') = \underline{\theta}$. Since

each initial teacher of a school has a different type, the pointing order can be arbitrary. For new teachers, assume that the tie-breaker ranks t above t' so that $t \vdash t'$. At the beginning of Step 1 of SI-CC, using the pointing behaviors of the definition, we obtain the graph in Figure 1a.

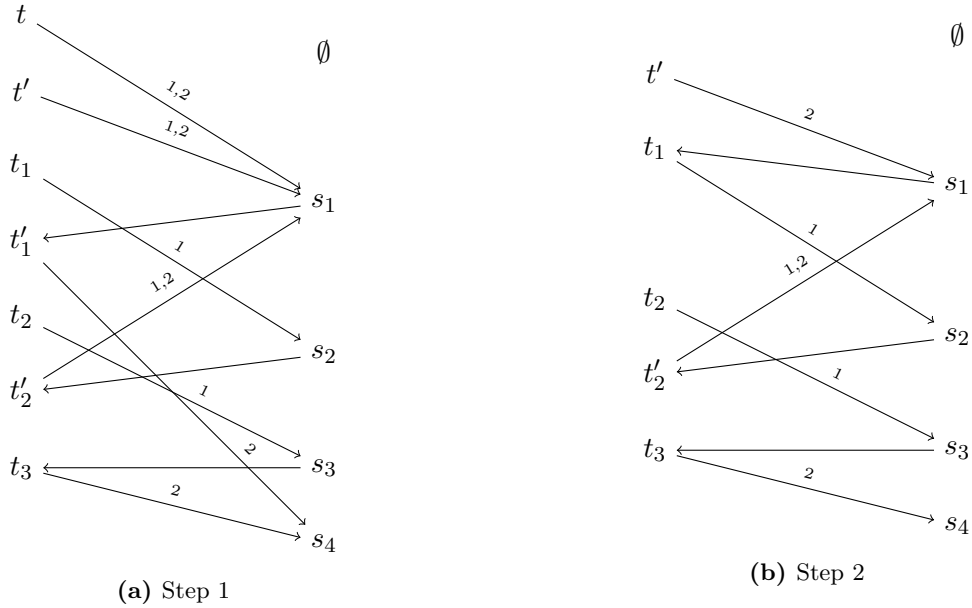


Figure 1: Graph of the steps 1 and 2 of SI-CC

For each arrow going from a teacher to a school, we report the improvement conditions, i.e. 1 and/or 2 in the definition of pointing rule for teachers, which hold for that arrow. One can note that there is no cycle in this graph. Thus there are only two possible chains: one starting at t or one starting at t' . Since $t \vdash t'$, we pick the one starting with t and, following the procedure described, implement the chain $\{t, s_1, t'_1, s_4\}$ since t'_1 points to s_4 only because of the improvement condition 2. At the beginning of Step 2, the graph becomes the one in Figure 1b.

In that case, one can check that the cycle $\{s_1, t_1, s_2, t'_2\}$ is implemented and that, at Step 3, the chain $\{t', s_1\}$ is implemented. At the beginning of Step 4, the graph of SI-CC is the one in Figure 2a. Note that even though teacher t_3 prefers s_4 to s_2 , he cannot point to the former because even though it has an empty seat left, there is no remaining new teacher.

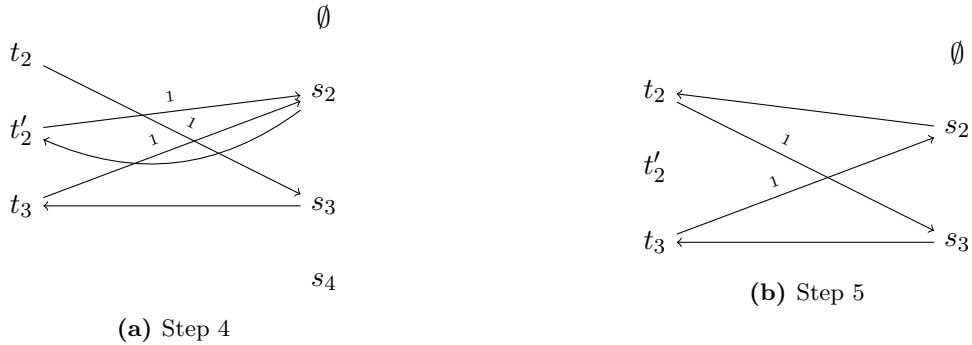


Figure 2: Graph of the steps 4 and 6 of SI-CC

In that step, we implement the cycle $\{s_2, t'_2\}$. At the next step, we obtain the graph in Figure 2b. Note that even though t_3 has a low experience and t_2 a high experience, the former can still point to s_2 since it has accepted t_1 , a high experience teacher, at step 2 so that the improvement condition 1 in the pointing rule of teachers is satisfied. So we implement the cycle $\{s_2, t_2, s_3, t_3\}$ and the algorithm stops.

Now we are ready to state our main result in this section.

Theorem 1 *The SI-CC mechanism is SI teacher optimal and strategy-proof.*

SI teacher optimality of the mechanism is delicate to show. Note that SI teacher optimality implies that the outcome matching is Pareto undominated for teachers among all status-quo improving matchings. However, the pointing rule of teachers has restrictions imposed by the school improvement conditions. That is, a teacher cannot arbitrarily point to the best school she likes. We show that the restrictions imposed by these conditions are the necessary and sufficient conditions for keeping status-quo improvement for schools without affecting the outcome being Pareto undominated for teachers. Therefore, implementing any further Pareto improvement for teachers would make the schools worse off with respect to the status quo. Moreover, imposing further restrictions for teacher pointing would prevent SI teacher optimality.

Strategy-proofness of the mechanism relies on several observations: First, once a teacher is pointed by a school, she will continue to be pointed until she is assigned. We show that the opportunity set for each teacher t , A_k^t , weakly shrinks across steps k . Although Improvement Conditions 1 or 2 may stop holding for a school via a teacher t across steps, we show that teacher t cannot affect which schools leave and stay in A_k^t before she is assigned by submitting different preferences.

An immediate corollary to the theorem is that the SI-CC mechanism is also (two-sided) Pareto efficient by Proposition 1.

Under the pointing rule of schools, in each step of SI-CC, each school points to one of its employee who has the lowest ranked type. One can think whether Theorem 1 holds when we consider alternative school pointing rules under SI-CC mechanism. Example 2 shows that SI-CC can be manipulated by a teacher and it is not SI teacher optimal under an alternative pointing rule.

Example 2 *Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be*

$$\omega_s = \{t_1, t_2\}, \omega_{s'} = \{t_3\}, \omega_{s''} = \{t_4\},$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$ and $\tau(t_1) \triangleright_s \tau(t_2) = \tau(t_3) = \tau(t_4)$. The preferences of the teachers are

$$s P_{t_1} s' P_{t_1} s'' P_{t_1} \emptyset,$$

$$s' P_{t_2} s P_{t_2} s'' P_{t_2} \emptyset,$$

$$s P_{t_3} s' P_{t_3} s'' P_{t_3} \emptyset,$$

$$s'' P_{t_4} s P_{t_4} s' P_{t_4} \emptyset.$$

If in the first step of SI-CC school s points to t_1 , the best school t_3 can point is s' . Therefore, she will be assigned to s' . In particular, under true preferences SI-CC assigns all employees to their status-quo schools. This outcome is not SI teacher optimal because it is Pareto dominated by another status-quo improving matching ν for teachers where $\nu(t_1) = \nu(t_3) = s$, $\nu(t_2) = s'$ and $\nu(t_4) = s''$. Moreover, if t_3 swaps the rankings of s' and s'' , then SI-CC selects ν , i.e., t_3 manipulates SI-CC when s points t_1 in the first step.

We would like to emphasize one possible generalization of SI-CC through school pointing rule. We can easily use the first school improvement condition given in the definition of SI-CC to dynamically update the school pointing rule such that monotonicity of opportunity set for teachers is preserved.

4 Stability, Choice Rule Design, and Status-quo Improving Deferred Acceptance

4.1 Status-quo Improving Stability

Although Pareto efficiency is a very appealing property of matchings, many real-life applications use fairness or stability notions, which often conflict with SI teacher optimality and in general with Pareto efficiency under incentive compatibility constraints. For example, in the French teacher reassignment application, the mechanism currently used is not Pareto efficient, while it satisfies a stability condition that is not necessarily status-quo improving.

To this end, we also introduce a stability concept that is consistent with status-quo improvement under a mild assumption about the number of new teachers in a market. Our notion has different requirements than Gale-Shapley stability (Gale and Shapley, 1962), which is extensively used in the literature, because in our setting we have a non-empty status-quo matching while most of the literature focuses on an empty matching as the status quo.

Consider a market P .

To introduce stability, we first start with blocking by an agent and a pair. A matching μ is **blocked by a teacher** t if $\emptyset P_t \mu_t$. A matching μ is **blocked by a school** s if there exists $t' \in \mu_s$ with $\theta_{\emptyset} \triangleright_s \tau(t')$. Observe that a status-quo improving matching is not blocked by any agent, while a matching that is not blocked by any agent may not be status-quo improving.

Given a teacher t and school s , a matching μ is **blocked by pair** (t, s) **through** $t' \in \mu_s$ if (i) $s P_t \mu_t$ and (ii) $\tau(t) \triangleright_s \tau(t')$. Similarly, a matching μ is **blocked by pair** (t, s) **through an empty slot** if (i) $s P_t \mu_t$, (ii) $\tau(t) \triangleright_s \tau(t')$ and (iii) $|\mu_s| < q_s$.

A matching μ is **Gale-Shapley stable** if there is no blocking agent and no blocking pair. This classical concept potentially conflicts with our most basic property, status-quo improvement:

Proposition 2 *A Gale-Shapley stable matching always exists, however, it may not be status-quo improving. Thus, a Gale-Shapley stable and status-quo improving matching may not exist.*

One may think that the cause of incompatibility of status-quo improvement with Gale-Shapley stability is not giving employment rights to teachers at their status-quo schools. Indeed the current system in France uses a strategy-proof mechanism that satisfies the following stability concept implicitly.²⁰

A matching μ is **teacher-status-quo-improving (teacher-SI) stable** if there is no blocking agent and no blocking pair through an empty slot, and if there is a blocking pair (t, s) through $t' \in \mu_s$, then $t' \in \omega_s$. This concept ignores blocking pairs as long as assigning the teacher to the school in the blocking pair would displace a status-quo employee of the school. This concept still does not resolve the main problem.

Proposition 3 *Even when there are no empty seats at schools at status quo and there are no new teachers, the current French mechanism is teacher-SI stable but not status-quo improving, while the status-quo matching is both teacher-SI stable and status-quo improving. Moreover, if there are empty seats at some schools at status quo, then a teacher-SI stable and status-quo improving matching may not exist.*

We should strengthen no blocking by an agent to status-quo improvement. Given the above impossibility result, however, this remedy alone does not resolve our non-existence problem when there are empty seats at the status quo. Instead, we introduce the following concept which implicitly gives new teachers rights over status-quo empty seats of a school.

A matching μ is **status-quo improving stable (SI-stable for short)** if

1. it is status-quo improving, i.e., $\mu_s \succ_s \omega_s$ and $\mu_t P_t \omega_t$ for all $s \in S$ and $t \in T$;
2. there is no blocking pair (t, s) through an empty slot; and
3. there is no blocking pair (t, s) through t' such that either $t, t' \in N$ or $t, t' \in T \setminus (N \cup \omega_s)$.

SI-stability requires status-quo improvement, which implies elimination of individual blocking, and elimination of blocking pairs through empty slots. Moreover, it requires elimination of any blocking pair (t, s) through t' such that t and t' are either new teachers or are currently employed by another school s' .

As a result, this concept is neither weaker (because of the more stringent individual blocking condition) nor stronger (because of the less stringent pairwise blocking conditions) than both Gale-Shapley and teacher-SI stability concepts.

Although it may appear counter intuitive to allow certain blocking pairs, it turns out that this is necessary to sustain status-quo improvement. The solution provided in the following subsection eliminates further blocking pairs such as the ones including new teachers through an existing teacher.

²⁰To make the current French setup more consistent with ours one may think that each teacher has a different type and the type ranking of each school is given by the government-dictated strict priority order used in France.

In Appendix B, by using examples we show that allowing the other blocking pairs not captured by Condition 3 is needed to guarantee existence of SI-stable matching.

4.2 Auxiliary Choice Rule Design and Status-quo Improving Deferred Acceptance Mechanism

In this section, we introduce a strategy-proof mechanism that is SI-stable under a mild assumption we will introduce below. Our main contribution here is to introduce an auxiliary choice rule for schools that will achieve SI-stability and strategy-proofness when it is used in conjunction with the teacher proposing deferred acceptance algorithm of Gale and Shapley (1962) adopted for complex matching terms by Roth and Sotomayor (1990) (which was itself adopted from more complex versions of such processes in Kelso and Crawford (1982), Roth (1984), Blair (1988)).

Given a school s , a **choice rule** is a function $C_s : 2^T \rightarrow 2^T$ such that for any $\hat{T} \subseteq T$, (i) $C_s(\hat{T}) \subseteq \hat{T}$ and $|C_s(\hat{T})| \leq q_s$.

Using the choice rules we will design below we will employ the well-known teacher proposing deferred acceptance algorithm. We consider the sequential version of this algorithm also known as the cumulative offer process (Hatfield and Milgrom, 2005) for more complex contractual matching terms:

Definition 2 *Teacher Proposing Deferred Acceptance Algorithm (DA):*

Step 1: Some teacher t' proposes to her favorite acceptable school, denoted by s' , if such a school exists. In this case, define $B_{s'}^2 \equiv \{t'\}$ and $B_s^2 \equiv \emptyset$ for each school $s \neq s'$. Otherwise, define $B_s^2 \equiv \emptyset$ for each school s .

Each school s holds teachers in $C_s(B_s^2)$ and rejects all other teachers in B_s^2 .

In general,

Step $k > 1$: Some teacher t'' who is not currently held by any school proposes to her most favorite acceptable school that has not rejected her yet, denoted by s'' , if such a school exists. In this case, define $B_{s''}^{k+1} \equiv B_{s''}^k \cup \{t''\}$ and $B_s^{k+1} \equiv B_s^k$ for each $s \neq s''$. Otherwise, define $B_s^{k+1} \equiv B_s^k$ for each school s .

Each school s holds $C_s(B_s^{k+1})$ and rejects all teachers in $B_s^{k+1} \setminus C_s(B_s^{k+1})$.

The algorithm terminates when each teacher is either rejected by all of her acceptable schools or currently held by some school. We assign each school the students it is holding.

Our main contribution in this subsection is the construction of an auxiliary choice rule for each school. Fix a school s . First, we need some additional concepts.

A **tie breaker** is a linear order over teachers \vdash as before. We construct a new linear order over the teachers in ω_s denoted by \blacktriangleright_s as follows:

For any $t, t' \in \omega_s$,

$$t \blacktriangleright_s t' \iff \tau(t) \triangleright_s \tau(t') \text{ or } [\tau(t) = \tau(t') \text{ and } t \vdash t'].$$

Observe that a better-type teacher is prioritized over a worse-type teacher, and when two same-type teachers are compared, then we use the tie breaker to prioritize one over the other.²¹

The auxiliary choice rule will use a lexicographic decision structure within a school by dividing the school into independent slots where each **slot** eventually represents a seat at the school. Such a model was previously introduced by Kominers and Sönmez (2016) in one-sided priority-based matching context for more complex contractual matching terms.

We fix a school s in this construction. Let $\mathcal{S}_s = \{s^1, s^2, \dots, s^{q_s}\}$ be the set of slots at school s . Without loss of generality we label the types in Θ as $\theta_1, \dots, \theta_{|\Theta|}$ based on the type ranking of the school such that $\theta_k \triangleright_s \theta_{k+1}$ for all $k \in \{1, \dots, |\Theta| - 1\}$. We define a ranking for each slot over $T \cup \{\emptyset\}$ where \emptyset denotes keeping the slot unfilled. The **ranking of slot** s^k , \succ_s^k , is defined separately for the slots representing the filled seats at the status-quo matching, i.e., for $k \leq |\omega_s|$, and slots representing the empty seats at the status-quo matching, i.e., for $|\omega_s| < k \leq q_s$:

- For filled slots s^k at the status quo, i.e., all $k \leq |\omega_s|$:
 - the teacher $t \in \omega_s$ who is ranked k 'th under \blacktriangleright_s has the highest ranking under \succ_s^k ,
 - any teacher t' with $\tau(t) \triangleright_s \tau(t')$ is ranked below \emptyset under \succ_s^k , and
 - the rest of the ranking under \succ_s^k is determined according to \triangleright_s such that ties between same type teachers are broken according to tie breaker \vdash .
- For empty slots s^k at the status quo, i.e., all k such that $|\omega_s| < k \leq q_s$:
 - a teacher t is ranked above \emptyset under \succ_s^k if and only if she is acceptable, i.e., $\tau(t) \triangleright_s \theta_\emptyset$,
 - any acceptable new teacher t (i.e., $t \in N$ and $\tau(t) \triangleright_s \theta_\emptyset$) is ranked under \succ_s^k above any teacher t' employed at status quo by some school (i.e., $t' \notin N$), and
 - the rest of the ranking under \succ_s^k is determined according to \triangleright_s such that ties between same type teachers are broken according to tie breaker \vdash .

Thus, the set of acceptable teachers for slot s^k is a superset of the set of acceptable teachers for slot s^{k-1} .

We will make the following mild overdemand assumption in the rest of this section involving new teachers and schools with excess status-quo capacity:

Assumption 1 *There exists a subset of new teachers $N' \subseteq N$ such that (i) there are at least as many new teachers in N' as empty seats at status quo, i.e., $|N'| \geq \sum_{s \in S} (q_s - |\omega_s|)$, and (ii) each teacher $t \in N'$*

- *considers all schools with excess capacity acceptable, i.e., if $q_s > |\omega_s|$, then $s P_i \emptyset$, and*
- *is acceptable for all schools with excess capacity, i.e., if $q_s > |\omega_s|$, then $\tau(t) \triangleright_s \theta_\emptyset$.*

In the absence of either part of Assumption 1, we can come up with examples such that some schools end up with fewer teachers than what they have under the status-quo matching and status-quo improvement is violated for schools (see Appendix B).

²¹Thus, linear order \blacktriangleright_s effectively reverses the ordering of status-quo employees of different types in the pointing order \triangleright_s used in the SI-CC mechanism, while it respects the school's FOSD preferences.

Since the auxiliary choice rule is defined through filling each slot one a time, we need to determine in which order the slots are processed. We process the slots in the natural order²²

$$s^1, s^2, \dots, s^{q_s}.$$

Definition 3 The *auxiliary choice rule* C_s of school s is defined through an iterative procedure. The auxiliary chosen set from the set of teachers \hat{T} by school s , denoted by $C_s(\hat{T})$, is determined as follows:

- **Step 1:** The most preferred acceptable teacher under \succ_s^1 in $\hat{T}_1 = \hat{T}$ is assigned to slot s^1 and she is removed. If there is no such teacher, then s^1 remains empty. Denote the remaining teachers with \hat{T}_2 .
In general,
- **Step $k \geq 2$:** The most preferred acceptable teacher under \succ_s^k in \hat{T}_k is assigned to slot s^k and she is removed. If there is no such teacher, then s^k remains empty. Denote the remaining teachers with \hat{T}_{k+1} .

The process terminates when all slots are processed, i.e., step q_s is the last step. Auxiliary chosen set $C_s(\hat{T})$ is the set of teachers assigned to the slots of school s .

We illustrate how an auxiliary chosen set is found in the following example.

Example 3 Suppose there are five teachers one of whom is new: $T = \{t_1, t_2, t_3, t_4, t_5\}$ and $t_5 \in N$. The status-quo match of school s , which has capacity $q_s = 3$ is $\omega_s = \{t_1, t_2\}$. The type ranking of school s is

$$\tau(t_1) = \tau(t_3) \triangleright_s \tau(t_2) \triangleright_s \tau(t_4) \triangleright_s \tau(t_5) \triangleright_s \theta_\emptyset.$$

The slot set of s is $\mathcal{S}_s = \{s^1, s^2, s^3\}$ such that s^1 and s^2 correspond to filled seats at status quo and s^3 corresponds to the empty seat.

Let the tie breaker \vdash be such that $t_1 \vdash t_3$.

We construct the rankings for each slot as follows:

$$\begin{aligned} t_1 \succ_s^1 t_3 \succ_s^1 \emptyset \succ_s^1 t & \quad \text{for any } t \notin \{t_1, t_3\}, \\ t_2 \succ_s^2 t_1 \succ_s^2 t_3 \succ_s^2 \emptyset \succ_s^2 t & \quad \text{for any } t \notin \{t_1, t_2, t_3\}, \\ t_5 \succ_s^3 t_1 \succ_s^3 t_3 \succ_s^3 t_2 \succ_s^3 t_4 \succ_s^3 \emptyset. & \end{aligned}$$

Suppose $\hat{T} = \{t_2, t_3, t_4, t_5\}$. Then, the auxiliary set of chosen teachers $C_s(\hat{T})$ is found as follows:

- **Step 1:** Teacher t_3 is the most preferred for slot s^1 among the teachers in $\hat{T}_1 = \hat{T}$. Hence, t_3 is assigned to slot s^1 and she is removed. We set $\hat{T}_2 = \hat{T}_1 \setminus \{t_3\}$.
- **Step 2:** Teacher t_2 is the most preferred for slot s^2 among the teachers in \hat{T}_2 . Hence, t_2 is assigned to slot s^2 and she is removed. We set $\hat{T}_3 = \hat{T}_2 \setminus \{t_2\}$.

²²Later we will explain why this precedence order is chosen.

- **Step 3:** Teacher t_5 has the highest priority for slot s^3 among the teachers in \hat{T}_3 . Hence, t_5 is assigned to slot s^3 and she is removed. We set $\hat{T}_4 = \hat{T}_3 \setminus \{t_5\}$.

Hence, $C_s(\hat{T}) = \{t_3, t_2, t_5\}$.

We define the following notions for the choice rules that will be crucial for our mechanism to be both strategy-proof and SI-stable.

The auxiliary choice rule C_s satisfies **substitutes** (Kelso and Crawford, 1982) if for all $\bar{T} \subseteq T$ and distinct teachers $t, t' \in \bar{T}$,

$$t \in C_s(\bar{T}) \implies t \in C_s(\bar{T} \setminus \{t'\}).$$

The auxiliary choice rule C_s satisfies the **law of aggregate demand** (Alkan and Gale, 2003, Hatfield and Milgrom, 2005) if for all $\bar{T}, \hat{T} \subseteq T$,

$$\bar{T} \subseteq \hat{T} \implies |C_s(\bar{T})| \leq |C_s(\hat{T})|.$$

Next, we show that C_s satisfies these two properties.

Proposition 4 *The auxiliary choice rule C_s satisfies the substitutes and law of aggregate demand conditions.*

We refer to the mechanism that selects the outcome of the DA algorithm using the auxiliary choice rules $(C_s)_{s \in S}$ that we designed as the **status-quo improving deferred acceptance (SI-DA)** for short) mechanism. The logic behind naming will be clear with the following result:

Theorem 2 *SI-DA mechanism is strategy-proof, and under Assumption 1, it is also SI-stable.*

Notice that, when there is no new teacher, i.e., $N = \emptyset$, Theorem 2 holds without Assumption 1.

In the proof of Proposition 4 the processing of slots does not play any role. Hence, the Proposition 4 holds for any order we use in the calculation of chosen teachers. As a result, SI-DA mechanism continues to be strategy-proof independent of the processing order of the slots. Moreover, the proof of SI-Stability of SI-DA does not rely on the processing order. Hence, SI-DA mechanism continues to be SI-Stable independent of the processing order of the slots. However, the processing order has an impact on the mobility and the welfare of the teachers.

Let \blacktriangleright_s and $\hat{\blacktriangleright}_s$ be arbitrary two processing orders of seats at school s such that $\hat{\blacktriangleright}_s$ is obtained from \blacktriangleright_s by swapping two adjacent slots s^k and s^ℓ where $k < \ell \leq |\omega_s|$. Let $\blacktriangleright_{\bar{s}} = \hat{\blacktriangleright}_{\bar{s}}$ for any $\bar{s} \neq s$. Let $D_{s'}$ and $\hat{D}_{s'}$ be the choice rules induced by $\blacktriangleright_{s'}$ and $\hat{\blacktriangleright}_{s'}$ by using the procedure defined in Definition 3 for all $s' \in S$. Let μ and $\hat{\mu}$ be the outcome of DA algorithm by using choice rules $D_{s'}$ and $\hat{D}_{s'}$ for all $s' \in S$. Then, the following proposition holds.

Proposition 5 *Each teacher t (weakly) prefers μ_t to $\hat{\mu}_t$.*

Proposition 5 implies that the processing order that we use for the first $|\omega_s|$ seats at each school s increases the welfare of the teachers compared to the alternative processing orders. Moreover, it also implies that the processing order that we use increases the teacher mobility. However, Proposition 5 does not say anything about the relative order the last $q_s - |\omega_s|$ seats at each school s . In the following example, we illustrate that we cannot find an optimal processing order for the last $q_s - |\omega_s|$ seats.

Example 4 Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3, t_4\}$, the status-quo matching be

$$\omega_s = \{t_4\}, \omega_{s'} = \{t_2\}, \omega_{s''} = \emptyset,$$

$q_s = 2$, $q_{s'} = q_{s''} = 1$, $\tau(t_1) \triangleright_s \tau(t_2) \triangleright_s \tau(t_3) \triangleright_s \tau(t_4)$, $\tau(t_1) \triangleright_{s'} \tau(t_3) \triangleright_{s'} \tau(t_4) \triangleright_{s'} \tau(t_2)$, and $\tau(t_1) \triangleright_{s''} \tau(t_2) \triangleright_{s''} \tau(t_3) \triangleright_{s''} \tau(t_4)$. The preferences of the teachers are

$$s P_{t_1} s' P_{t_1} s'' P_{t_1} \emptyset,$$

$$s P_{t_2} s'' P_{t_2} s' P_{t_2} \emptyset,$$

$$s P_{t_3} s'' P_{t_3} s' P_{t_3} \emptyset,$$

$$s' P_{t_4} s'' P_{t_4} s P_{t_4} \emptyset.$$

If s_1^1 is filled before s_1^2 , then under DA t_1 and t_3 are assigned to s , t_2 is assigned to s'' and t_4 is assigned to s' . If s_1^2 is filled before s_1^1 , then under DA t_1 and t_2 are assigned to s , t_3 is assigned to s'' and t_4 is assigned to s' . Hence, we cannot have the same conclusion as in Proposition 5.

We use tie-breaking in the construction of the slot priorities. Inclusion of this exogenous tool causes efficiency loss: SI-DA does not select a Pareto undominated SI-stable matching (and therefore, it is not Pareto efficient either). We illustrate this situation in the following example.

Example 5 Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be

$$\omega_s = \{t_1\}, \omega_{s'} = \{t_2\}, \omega_{s''} = \{t_3\},$$

$q_s = q_{s'} = q_{s''} = 1$, $\tau(t_1) = \tau(t_2) = \tau(t_3)$, and all teachers are acceptable for all schools. The preferences of the teachers are

$$s' P_{t_1} s P_{t_1} s'' P_{t_1} \emptyset,$$

$$s P_{t_2} s' P_{t_2} s'' P_{t_2} \emptyset,$$

$$s' P_{t_3} s'' P_{t_3} s P_{t_3} \emptyset.$$

Let $t_2 \vdash t_3 \vdash t_1$ be the tie breaker, then the rankings of the slots are given as:

$$t_1 \succ_s^1 t_2 \succ_s^1 t_3,$$

$$t_2 \succ_{s'}^1 t_3 \succ_{s'}^1 t_1,$$

$$t_3 \succ_{s''}^1 t_2 \succ_{s''}^1 t_1.$$

SI-DA assigns t_1 to s , t_2 to s' and t_3 to s'' . However, this outcome is Pareto dominated by another status-quo improving stable matching in which t_1 , t_2 and t_3 are assigned to s' , s , and s'' , respectively.

5 Generalizing Lower Bound for Welfare Improvements for Schools

In this section, we relax the FOSD requirement for status-quo improvement. Let school s 's ranking over the types be $\theta_1 \triangleright_s \theta_2 \triangleright_s \dots \triangleright_s \theta_n \triangleright_s \theta_0$. That is, any type $\theta \notin \{\theta_1, \theta_2, \dots, \theta_n\}$ is unacceptable for school s . Let $d = (d_{\theta_1}, d_{\theta_2}, \dots, d_{\theta_n}) \in \mathbb{R}^n$ be **threshold acceptability vector** where

- $d_{\theta_1} \leq |\omega_s^{\theta_1}|,$
- $d_{\theta_1} + d_{\theta_2} \leq |\omega_s^{\theta_1} \cup \omega_s^{\theta_2}|,$
- ...
- $d_{\theta_1} + d_{\theta_2} + \dots + d_{\theta_n} = |\omega_s^{\theta_1} \cup \omega_s^{\theta_2} \cup \dots \cup \omega_s^{\theta_n}| = |\omega_s|.$

Notice that threshold values are relaxation over the number of current students from best to worst in a cumulative sense. A matching μ is **d-improving for school s** if all teachers in μ_s have acceptable types for school s and

- $d_{\theta_1} \leq |\mu_s^{\theta_1}|,$
- $d_{\theta_1} + d_{\theta_2} \leq |\mu_s^{\theta_1} \cup \mu_s^{\theta_2}|,$
- ...
- $d_{\theta_1} + d_{\theta_2} + \dots + d_{\theta_n} \leq |\mu_s^{\theta_1} \cup \mu_s^{\theta_2} \cup \dots \cup \mu_s^{\theta_n}| = |\mu_s|.$

A matching μ is **d-improving** if it is d-improving for all schools and no teacher t is assigned to a school worse than ω_t . Notice that, if matching μ status-quo improves ω , then μ is d-improving.

Under this generalization of the status-quo improvement lower bound, we can apply the SI-CC and teacher proposing DA mechanisms after we relabel the types of current employees for each school. In particular, for each θ_k with $k < n$ we select $|\omega_s^{\theta_k}| - d_{\theta_k}$ teachers from ω_s and treat them as type θ_n when we determine the pointing rule of school s and the pointing rule of teachers under SI-CC and priority rankings of seats under teacher proposing DA. For the rest of the steps of the mechanisms, each teacher is treated with her own type.

These mechanisms inherit their desired properties mentioned in Sections 3 and 6.5 under these modifications.

6 Empirical Analysis

This section provides empirical evidence on the changes that the mechanisms we suggest would bring in a real world setting: teacher assignment to regions in France. After presenting the institutional context and the data, we structurally estimate teachers preferences over regions, and run counterfactuals to quantify the improvements that our mechanisms may yield.

6.1 Institutional background

Teacher recruitment and assignment. Teacher certification and recruitment is highly centralized in France. Anyone who wishes to become a teacher has to pass a competitive examination. Those who succeed are allocated a teaching position by the ministry for a probation period of one year, at the end of which they get tenure or not. Once they get tenure, teachers in public schools become civil servants. The government manages both the first assignment of newly tenured teachers to a school, and the mobility between schools of tenured teachers who previously received an assignment but wish to change.²³

Mobility request, vacant positions and newly recruited teachers. We use data on the assignment of teachers to one of the 31 French regions in 2013 for our empirical analysis. There were 700,000 secondary public school teachers in France that year, a number that fluctuates from year to year due to both entries and exits from the profession. Exits are mainly due to teachers retiring—9,793 public secondary school teachers retired in 2013—while entries correspond to newly recruited teachers who have passed the recruitment exam and validated their probation year. As a result, when organizing the annual mobility process, the central administration has to take into account a large pool of tenured teachers who already have a position and wish to change, but also some vacant positions that need to be filled, and some newly recruited teachers who ask for their first assignment. In 2013, the year for which we have data, about 25,100 teachers took part in the centralized regional mobility process. Among them 17,200 are tenured teachers and 7,900 are newly recruited teachers who need a first assignment.

A two-step assignment process. The assignment procedure takes place in two successive steps.²⁴ During the first step, which is managed centrally by the ministry, teachers are assigned to one of the 31 French regions using a first algorithm. In the second step, teachers newly assigned to a region and teachers who wish to change schools within their region submit a list of ordered schools. Since 1999, this step is managed directly by local administrations within the regions. Our empirical analysis focuses on the first regional assignment because of potential strategic reports of preferences during the second phase.²⁵²⁶ Participation to the assignment mechanism is compulsory for all newly tenured teachers who have never been assigned a position. Participation is optional for other tenured teachers who are never forced to change region or school.

²³This centralized assignment process is used for public school teachers only. Private schools make up 16% of teachers. For them, the recruitment process is similar but public and private school teachers face completely different rules for their mobility—between regions and between schools. In private schools, teachers apply directly to schools—as would be the case in usual labor markets.

²⁴Before 1999, teachers' assignment to schools was managed centrally by running an algorithm once, which assigned teachers directly to schools. This highly centralized process was argued to be at odds with the regional nature of most demands: the majority of teachers asking for a transfer ranked schools within their current regions.

²⁵Preferences reported during the second phase of the assignment are more difficult to interpret for two reasons. First, teachers can only rank up to 20 or 30 schools, depending on the region. Second, in addition to rank schools, teachers can also rank larger geographic areas, such as cities for instance.

²⁶Of course, even for teachers assignment to regions, one may wonder if preferences over regions are well-defined objects since what matters for teachers is their assigned school within a region. Combe, Tercieux and Terrier (2020) provide a detailed discussion of this and supportive evidence, notably on teachers lexicographic preferences.

6.2 Data and Descriptive Statistics

We use data on the assignment of teachers to one of the 25 French regions in 2013.²⁷ We have information on teachers' reported preferences and their initial assignment (if any), the priorities of the regions, and the number of vacant positions in each region. We keep all teachers from the 8 largest subjects, such as French, math, English, and Sports. We discard couples from the sample because they benefit from a specific treatment in the assignment process.²⁸ Finally, in order to keep the market structure balanced, we drop one region seat for each teacher we omit.²⁹ Our final sample contains 10,460 teachers: 5,833 initially assigned teachers (55.8%) and 4,627 new teachers. Table A.1 shows the decomposition by subject.

A central motivation of our analysis is to rebalance the unequal distribution of teachers across regions. Part of this large imbalance stems from differences in regions attractiveness. Table 1 reports descriptive statistics on teachers (Panel A), their initial assignment (Panel B), and the region they rank first (Panel C).³⁰ Two regions surrounding Paris, called Créteil and Versailles, appear as particularly unattractive. The imbalance is blatant when comparing the number of teachers asking to leave the region and the number asking to enter. For instance, in math, 52.3% of the tenured teachers who ask to change region come from Créteil or Versailles, but only 3.4% rank one of these two regions as their first choice.³¹ Table A.2 provides additional evidence on attractiveness differences and its potential determinants for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Créteil, Versailles, and Amiens), and three intermediate regions (Paris, Aix-Marseille, and Grenoble).³²

The large share of mobility requests that originate from unattractive regions has a direct consequence on the annual mobility flows: Under the current assignment system, a large number of teachers exit these regions, which results in numerous vacant positions that need to be filled. As a mechanical result, about 50% of the newly recruited teachers get their first assignment in one of the three least attractive regions (Créteil, Versailles, and Amiens). This structural imbalance is a serious concern for policy makers. It is frequently raised as a reason for the lack of attractiveness of the teaching profession in France.³³ In addition, it creates large differences in the age profile of

²⁷We discard the 6 overseas regions because of their specificities in terms of (i) teacher preferences—in contrast to what we find in our estimates, distance from the current location often becomes an attractive feature—and (ii) regions priorities—some of these regions, like Mayotte, give teachers who grew up in these regions a bonus of points when they rank it first.

²⁸Spouses in two different subjects can submit joint mobility applications (by submitting two identical lists). This creates dependencies across markets for different fields.

²⁹For each tenured teacher we discard, we dropped his corresponding position. For new teachers, we count the number N of teachers discarded. Then we compute the share S of vacant positions that this number represents, and we delete $S\%$ of the vacant positions in each region.

³⁰Appendix E.1 provides a detailed description of each variable.

³¹This rate is a bit larger for new teachers. Between 10 and 15% of them rank Créteil or Versailles as their first choice across the 8 subjects we consider in our analysis.

³²Attractiveness is measured as the ratio of the number of tenured teachers asking to enter a region over the number of teachers asking to leave the region. This ratio ranges from 15.5 in Rennes to 0.03 in Créteil.

³³The most common recruitment exam in France is called the CAPES. Every year, the ministry decides on the number of teaching positions it opens for the exam. In 2014, 24% of the CAPES positions remained vacant because of both a lack of applicants and the poor quality of those applying. The shortage of teachers has not improved since.

Table 1: Descriptive Statistics on Teachers and Regions

	Teachers with initial assignment (Tenured)			Teachers without initial assignment (New)		
	French (1)	Math (2)	English (3)	French (4)	Math (5)	English (6)
Panel A. Teachers' characteristics						
% Female	76.1	47.0	85.4	80.3	41.7	80.4
% Married	48.5	45.0	46.8	41.1	39.4	40.9
% In disadvantaged school	10.4	13.2	4.4	0.0	0.0	0.0
Experience (in years)	7.48	7.23	7.18	2.76	2.24	2.30
% Advanced teaching qualif	7.9	29.1	8.8	16.8	31.7	15.2
Panel B. Characteristics of the region teachers are initially assigned to						
Is birth region	8.7	8.6	9.3	-	-	-
Is Créteil or Versailles	37.7	52.3	35.6	-	-	-
Is in South of France	5.6	9.3	12.7	-	-	-
% students in urban area	61.7	67.4	64.0	-	-	-
% disadvantaged students	52.5	54.0	53.5	-	-	-
% students in priority educ	26.0	24.5	22.7	-	-	-
% students private school	15.2	16.3	17.4	-	-	-
% teacher younger than 30	11.9	13.3	11.3	-	-	-
Panel C. Characteristics of the region teachers rank first						
Distance to init region (km)	2148.7	1316.9	1521.9	-	-	-
Is birth region	36.5	35.8	40.0	35.8	38.4	39.3
Is in South of France	25.2	25.2	25.4	20.1	18.6	20.0
Is Créteil or Versailles (CV)	2.8	3.3	3.4	14.3	11.9	12.5
% students in urban area	60.2	56.2	51.7	61.9	58.6	59.2
% disadvantaged students	52.8	53.1	53.5	53.3	53.1	53.0
% students in priority educ	20.3	19.9	17.9	21.8	20.6	21.0
% students private school	23.7	22.9	25.8	22.2	21.8	21.7
% teacher younger than 30	6.5	6.4	6.0	8.2	8.1	8.2
Observations	859	605	628	786	958	746

Notes: This table reports descriptive statistics for teachers and regions in three subjects: French, math and English. Statistics are reported for the sample of teachers we use for the demand estimations. Columns (1) to (3) report statistics for teachers with an initial assignment (referred to as “tenured” in the text). Columns (4) to (6) report statistics for teachers without an initial assignment (referred to as “new teachers” in the text). New teachers have missing values for statistics related to the region of initial assignment. We discard teachers who submit a joint list with their partner, teachers who are from one of the six regions that are overseas, and teachers for whom one of the individual characteristics is missing. The last row reports the number of teachers in each subject. Panels A, B, and C respectively present descriptive statistics on teachers, on the region in which they are initially assigned, and on the region they rank first.

teachers across regions. As reported in column (3) of Table A.2, the ratio of the number of teachers aged more than 50 over the number of teachers aged less than 30 is equal to 1.1 and 1.6 in Créteil and Versailles. In contrast, the most attractive region of Rennes had 8 times more teachers aged 50 and above than teachers aged 30 and below. In Bordeaux and Toulouse, it was 6.5 and 5.3 times more.

Several papers have found that teachers tend to make their students progress less during the first years of their career than when they have more experience (Chetty, Friedman and Rockoff (2014) Rockoff (2004)). Reducing the unequal distribution of teachers across regions and reducing the chances to be assigned a disadvantaged region in the first place became one of the objectives of the French policy makers, who see this as a way to both reduce achievement inequalities between students and improve the attractiveness of the teaching profession in the longer-run.

6.3 Specifications of the Empirical Analysis

Algorithms. The theoretical section shows that both the SI-CC and SI-DA algorithms can improve the distributions of teachers in schools. Our counterfactual analysis aims to quantify the performance of these algorithms but also to benchmark them with two algorithms:

- **Benchmark for SI-CC:** A version of SI-CC (called “TTC*”) that accounts for teachers types but relaxes the mechanism features that ensure status-quo improvement. More precisely, this mechanism differs from SI-CC in two respects: (1) we lift the restrictions on the set of schools (noted A_t^k) that a teacher can point to, and (2) tenured teachers can now start a chain (and potentially leave their position without being replaced). This benchmark is close to the well-known mechanism You Request My House-I Get Your Turn (YRMH-IGYT) with existing tenants (Abdulkadiroğlu and Sönmez, 1999, Sönmez and Ünver, 2009) which is strategy-proof, Pareto-efficient and individually rational for teachers but is not status-quo improving. Intuitively, TTC* might therefore be expected to generate more mobility than our mechanisms, but at the cost of a potentially more unequal distribution.
- **Benchmark for SI-DA:** A version of SI-DA (called “DA*”) that accounts for teachers types but relaxes the mechanism features that ensure status-quo improvement.³⁴ More precisely, this mechanism differs from SI-DA in two respects: (1) we lift the restrictions on schools priorities (i.e an applicant teacher with a less-preferred type than a status-quo teacher will no longer be considered as unacceptable by a school), and (2) vacant positions in a region no longer prioritize new teachers over tenured teachers. In practice, this benchmark is equivalent to the algorithm currently used by the French ministry of education—called DA*, see Appendix F for a formal description—with one exception: we replace the schools priorities by those that account for teachers types. Incorporating teachers types into this mechanism provides an interesting benchmark that targets (weak) stability, potentially at the cost of efficiency and distributional objectives.

³⁴We borrow the notation DA* to Combe et al. (2020) who study a modified version of DA which ensures status-quo improvement for teachers. We kept the same logic for our notation when dealing with TTC-based mechanisms using TTC*.

Priorities and Types. We run our different algorithms using as inputs teachers preferences, their types, and regions priorities over teachers. To illustrate the theory, we define a teacher type as her experience and we classify teachers into 12 experience bins, where the first bin corresponds to teachers with 1 or 2 years of experience, the second bin to teachers with 3 to 4 years of experience, and so on... A large number of teachers belong to the first bin. To more finely distinguish teachers by experience, we further decompose the first bin by ordering newly tenured teachers above tenured teachers. To define regions’ rankings over teachers’ types, we start by identifying which regions would benefit most from receiving more experienced teachers. To do so, we compute teachers average type in each region (see Figure A.1) and we classify regions into two groups based on whether their average type is above or below the median of the regions types. The first group contains all regions whose average type is strictly below the median, i.e younger regions that could benefit from receiving more experienced teachers. We set the priorities so that these regions rank types by decreasing levels of experience, i.e the most experienced teachers are always preferred to the least experienced teachers. The second group contains all regions whose average type is above the median. These regions rank types by increasing levels of experience. Our ultimate goal when we set “schools’ preferences” is to ensure that the assignment picked by our algorithms eventually yield a more equal distribution of teachers across regions. Of course, the way in which we set types is tailored to one of the main objectives of the French policy makers (see, for instance, Section 6.2). Our goal is to illustrate how our mechanisms perform—and understand the implications of imposing status-quo improvement—in such a context where the main goal is to achieve a more even distribution of teachers in terms of experience.

We also assume that all regions find all types acceptable. This means that any teacher is always preferred to a vacant position.³⁵ Finally, running SI-CC (and its variants) requires an additional ordering over teachers to determine which chain will be selected.³⁶ For the main results presented in the paper, we use the true point system of the French ministry and sort teachers by decreasing level of the maximum points they obtain over all regions. However, we show in an Appendix D that the performance of the SI-CC mechanism is sometimes sensitive to the ordering chosen. We report robustness results in which we flip the ordering to rank the teachers by increasing level of their maximum priority points.

6.4 Structural Estimation of Teachers’ Preferences

Teachers can rank all regions when they submit their preference list and the ministry uses a modified version of the deferred acceptance algorithm to assign teachers to regions, which means that it is a dominant strategy for teachers to be truthful. Yet, even under strategy-proof mechanisms, a number of papers show that truthfulness is a strong assumption (Chen and Sönmez, 2006, Pais and Pinter, 2008, Rees-Jones, 2018, Chen and Pereyra, 2019, Hassidim, Marciano, Romm and Shorrer,

³⁵This is a natural assumption since, in practice, all teachers are acceptable for all regions. For tie-break, we need an additional ordering over teachers for the SI-CC, SI-DA, and two benchmark algorithms. We use the tie-breaking rule used by the French ministry which uses the date of birth of teachers and some extra conditions for the rare cases with the same date of birth.

³⁶This ordering only applies to new teachers under SI-CC. It applies to all teachers under “TTC*”.

2017). In our context, French teachers have reasonably accurate information on their acceptance probabilities in each region, which might encourage some teachers to discard regions where their chances to be accepted are too low.³⁷ These omissions could introduce a bias in any counterfactual analysis done using teachers’ reported preferences. Combe, Tercieux and Terrier (2020) previously rejected truth-telling among French teachers. To avoid this potential bias, instead of using the reported preferences, we estimate teachers’ preferences using an identifying assumption (presented below) that does not require teachers to be fully truthful.

Model. We estimate teachers’ preferences over regions using the following utility function:³⁸

$$u_{t,R} = \delta_R + Z'_{t,R}\beta + \varepsilon_{t,R} \quad (5)$$

Teacher t ’s utility for region R is a function of region fixed effects δ_R , teacher-region-specific observables $Z_{t,R}$ (with coefficients β) and a random shock $\varepsilon_{t,R}$ which is i.i.d. over t and R and follows a type-I extreme value distribution, Gumbel(0, 1). The region fixed effect captures region characteristics such as average socio-economic and academic level of students in the region, cultural activities, housing prices, facilities, etc... We estimate preferences separately for teachers who have an initial assignment and those who do not. This allows us to include a richer set of variables for the former group. The vector $Z_{t,R}$ includes dummies specifying if the region is the birth region. For teachers with an initial assignment, it also includes a dummy for the region in which a teacher is currently assigned, as well as the distance between the region ranked and the current region of a teacher. $Z_{t,R}$ also includes interaction terms between teachers and schools characteristics (that are presented in Panels A and B of Table 1). We apply standard scale and position normalization, setting the scale parameter of the Gumbel distribution to 1 and the fixed effect of the Paris region to 0.

Identifying assumptions. To avoid the potential bias generated by teachers omitting regions they consider as unfeasible, we estimate teachers preferences under a weaker “stability assumption” developed by Fack, Grenet and He (2019) and applied to the teacher market by Combe, Tercieux and Terrier (2020).³⁹ We start by defining the *feasible set* of each teacher as the set of regions that have a cutoff—that is, the lowest priority of the teachers assigned to a region—smaller than his own score. These are regions a teacher could be assigned to if he was ranking the region first in his

³⁷Cutoffs values for entry in each region are published every year. Combe, Tercieux and Terrier (2020) show that these cutoffs are relatively persistent over time, so they provide reasonably accurate information to teachers on their chances to enter each region.

³⁸In the end what teachers primarily care about is the school they obtain within a region, which questions whether preferences over regions are well-defined objects. Combe, Tercieux and Terrier (2020) show evidence that teachers preferences seem to be lexicographic, i.e that teachers primarily care about the region in which the school is located and about a school within that region. The paper also shows that changes in a number of algorithms in the first phase (the regional assignment) only marginally impact the pool of participants in the second phase (the school assignment) so that teachers’ assessments of their school assignment probabilities (within a region) should not vary much between mechanisms. This implies that teachers’ preferences over regions may be relatively insensitive to the mechanism used.

³⁹Combe, Tercieux and Terrier (2020) provide an in-depth discussion of the two alternative identifying assumptions (truthfulness versus stability), as well as statistical tests in favor of the later. For more references on estimations that do not require truth-telling, see Akyol and Krishna (2017), Artemov, Che and He (2019), Agarwal and Somaini (2018), Calsamiglia, Fu and Güell (2020). They mainly focuses on the estimation of the preferences of tenured teachers and we use the same estimation in our analysis. Here, we provide an additional detailed discussion on the estimation of the preferences of new teachers.

rank order list. The key identifying assumption is that, for each teacher, the region obtained is his most preferred region among all regions that are in his feasible set.⁴⁰ Hence, we estimate a discrete choice model with personalized choice sets. Choice probabilities have closed form solutions and we estimate parameters using maximum likelihood.

Estimation results. Table 2 reports preference estimates for “tenured” teachers (i.e. those who have an initial assignment) and new teachers who do not have an initial assignment. We run the estimations in each of the eight subjects separately and report results for Math and French teachers in the table. The first nine rows report coefficients for a selected set of region fixed effects. They reveal an interesting difference between the preferences of tenured and new teachers. While the Créteil and Versailles regions are very unattractive for tenured teachers (as indicated by the negative coefficient of their fixed effect relative to the region of Paris), these regions are far more attractive for new teachers, who often see a first position in a disadvantaged school as a stepping stone for better positions in the future.⁴¹ The fact that Créteil and Versailles are more attractive for new teachers than for tenured teachers surely contributes to the unequal distribution of teachers denounced by policy makers. Yet, this is not the only explanation for teachers unequal distribution. The counterfactual analysis we present in the next section shows that the mechanism used also shapes the distribution of teachers in important ways. The fact that preferences alone are not driving the unequal distribution is fundamental for our ability to improve both teachers distribution and teachers welfare.

Fit measures. Our main fit measure considers the top two regions that a teacher has included in his submitted rank order list (ROL). We then compute the probability of observing this particular preference ordering in the ROL predicted with our estimations. This fit measure based on relative ranking (instead of the characteristics of the school ranked first for instance) is particularly adapted to our environment in which some teachers might not rank regions which they consider as unfeasible.⁴² In addition to the overall fit quality, we also compute fit measures for the teachers who come from the two least attractive regions of Creteil and Versailles (CV). Looking at fit quality for this sub-group of teachers is particularly important because teachers from CV represent a large share of the teachers who submit a mobility request every year and they are more likely to stay in their position than teachers from other regions. These two facts could affect the preference estimation for these teachers under our stability assumption. Across the 8 subjects, our fit measures range from 0.62 to 0.72 for tenured teachers and from 0.56 to 0.69 for new teachers, a fit quality which compares favorably to the one obtained by Fack, Grenet and He (2019) (between 0.553 and 0.615).

Simulations. We use our estimates of teachers preferences to draw their rank ordered list 1,000 times using Equation 5. After having drawn preferences, we keep the entire set of regions

⁴⁰This assumption is theoretically founded: Artemov, Che and He (2019) show that, in a large market environment, any (regular) equilibrium outcome of DA* must have this property.

⁴¹Teachers who stay in a disadvantaged school for at least 5 years benefit from additional priority when they ask to change region or school.

⁴²When teachers skip regions perceived as unfeasible, the first region they report might not be their preferred region—and indeed, the tests we perform reject truth-telling—but conditional on ranking schools, the order in which a teacher ranks the schools might correspond to a teacher true preferences. This is why we prefer to use a fit measure that is based on relative ranking than on the characteristics of the school ranked first for instance.

without imposing any truncation.⁴³ In each of the 8 subjects and for each preference draw, we use these simulated preferences and the priorities from our data to run the mechanisms. The results reported in next section correspond to averages over the 1,000 preference draws, aggregated over the 8 subjects.

⁴³The full set of regions contains the initial assignment for tenured teachers but, obviously, not for new ones. This implicit assumption that new teachers do not have an outside option is in line with the policy of the ministry. Teachers are indeed not required to rank all regions when they submit their lists, but the ministry fills the incomplete lists of new teachers to make sure that all of them get an assignment—even those who ranked few regions.

Table 2: Teachers Preference Estimates

	Tenured			New				
	French		Math	French		Math		
	coef (1)	s.e. (2)	coef (3)	s.e. (4)	coef (5)	s.e. (6)	coef (7)	s.e. (8)
Region BESANCON	-3.88***	(0.99)	0.37	(0.8)	1.32*	(0.66)	3.44***	(0.60)
Region BORDEAUX	-1.36	(0.95)	1.12	(0.66)	2.97***	(0.61)	2.69***	(0.46)
Region DIJON	-5.08***	(0.97)	-2.88***	(0.73)	1.71**	(0.59)	2.61***	(0.41)
Region LILLE	-5.09***	(0.95)	-1.55	(0.81)	1.58*	(0.62)	2.38***	(0.54)
Region REIMS	-6.22***	(1.00)	-3.60***	(0.74)	0.85	(0.62)	1.19*	(0.49)
Region AMIENS	-6.44***	(1.06)	-3.31***	(0.75)	1.32*	(0.61)	2.21***	(0.44)
Region ROUEN	-5.96***	(0.97)	-2.17**	(0.69)	1.54*	(0.60)	2.85***	(0.46)
Region CRETEIL	-6.66***	(1.00)	-3.65***	(0.71)	3.39***	(0.51)	3.56***	(0.36)
Region VERSAILLES	-5.12***	(0.89)	-2.13***	(0.60)	2.74***	(0.54)	3.32***	(0.37)
Current region	4.97	(6.72)	-15.72	(8.27)				
Birth region	10.21**	(3.41)	14.89***	(3.53)	7.61**	(2.31)	10.16***	(2.55)
Distance current region	-23.33***	(4.61)	-23.52***	(5.47)				
% stud urban x Current region	-5.82***	(0.85)	-5.40***	(1.15)				
% stud urban x Teach from CV	2.81***	(0.71)	0.12	(0.68)				
% stud in priority educ x Married	-7.61***	(1.60)	-3.89*	(1.65)	-10.77***	(1.46)	-11.54***	(1.21)
% stud in priority educ x Current region	11.26***	(2.99)	0.67	(3.80)				
% stud in private school x Teach in disadv sch	5.48**	(2.01)	6.58***	(1.83)				
% teachers younger than 30 x Advanced qualif	10.59**	(3.65)	0.24	(3.06)	5.56*	(2.77)	-2.64	(1.77)
% teachers younger than 30 x Current region	52.42***	(5.19)	54.15***	(6.47)				
% teachers younger than 30 x Birth region	-22.08***	(3.73)	-19.10***	(4.79)	-13.66***	(2.79)	-16.73***	(2.65)
Region in South of France x Teach from CV	-1.27***	(0.37)	0.35	(0.36)				
Number of teachers	859		605		669		958	
Fit measure	0.669		0.674		0.625		0.623	
Fit measure - Teach from CV	0.682		0.659		-		-	

† Notes: This table reports selected coefficients from estimations of teachers preferences for regions characteristics based on Equation 5. We use the estimation method recently developed by Fack, Grenet and He (2019). We use a discrete choice model with personalized feasible choice sets. For each teacher, his feasible choice set is the set of regions that have a cutoff smaller than his own score. We estimate parameters in columns 1, 3, and 5 using maximum likelihood. We set the fixed effect of the Paris region to 0. The last row reports our goodness of fit measure, that we compute by looking at the top two regions that a teacher has included in his submitted rank order list (ROL). We measure, for each teacher, the probability of observing this particular preference ordering in the ROL predicted with our estimations. We then average these probabilities across teachers. Stars correspond to the following p-values: * p < .05; ** p < .01; *** p < .001.

6.5 Relative Performance of the SI-DA and SI-CC mechanisms

We start by discussing the relative performance of SI-CC and SI-DA. While the former is SI teacher optimal—i.e. efficient for teachers among status-quo improving mechanisms—SI-DA is SI-stable—i.e. stable among status-quo improving mechanisms. Comparing the performance of these two mechanisms that target constrained-efficiency and constrained-stability is important. The mechanism that is currently used by the French ministry of education is teacher-SI stable, i.e. it is stable if one assumes that a teacher does not feel justified envy for another teacher who is initially matched to a school. This suggests that policy makers consider stability as an important feature of assignments. Yet, it is well known that stability often comes at the cost of efficiency. We investigate in this section whether a similar trade-off exists between SI-DA and SI-CC.

Validation of Assumption 1. We show in Section 4 that the SI-DA mechanism is SI-stable under the assumption that there are at least as many new teachers as empty seats in a market. Table A.1 shows that this assumption holds in each of the 8 subjects we consider. This means that the matching obtained under SI-DA is status-quo improving compared to the initial matching.

Distributions of teacher experience. We start by comparing, for different regions, the cumulative distribution of teachers' experience under SI-CC and SI-DA. We classify teachers into 12 experience bins.⁴⁴ Figure 1 shows the cumulative distribution of teachers' experience in the three youngest regions of France (Amiens, Versailles, and Créteil). These regions' teachers represent 78% of the disadvantaged regions teachers.⁴⁵ SI-DA slightly outperforms SI-CC in these regions, i.e. it assigns fewer inexperienced teachers. 1,040 teachers with one or two years of experience are assigned to one of the three regions under SI-DA versus 1,371 under SI-CC. SI-DA also improves the experience distribution in the three oldest regions by assigning them a larger number of inexperienced teachers.

Stability-efficiency trade-off. However, SI-DA's better distributional performance comes at a large cost in terms of teacher mobility. Table 3 shows that only 3,919 teachers obtain a new assignment under SI-DA, compared to 5,510 under SI-CC. The lack of mobility is particularly striking for tenured teachers. Only 7 of them move from their initial position, compared to 367 under SI-CC. The very low level of mobility under SI-DA is due to the strength of the requirement which jointly imposes status-quo improvement and stability. Indeed, status-quo improvement implies that many tenured teachers from unattractive regions will be unable to leave the region given that the strong requirement that the distribution of teachers' experience in these regions must increase (together with the low demand by tenured teachers for these regions). Given this, any teacher entering a region almost automatically generates justified envy from such a tenured teacher stuck in an unattractive region (in particular, since their relative low experience makes them high-priority for the old regions like Bordeaux). The stability requirement prevents such assignment from happening and, thus, blocks mobility.

⁴⁴To construct the cumulative distribution of teacher experience in each region, we consider vacant positions as the least preferred type (as shown on the right of the x-axis in Figure 1).

⁴⁵See Figure A.1 for the types distribution in regions.

This simple example and the results from our counterfactual analysis show that, under DA-based mechanisms that require stability, imposing a status-quo improvement can have the unintended consequence of dramatically blocking mobility. Said differently, prioritizing stability and status-quo improvement (under SI-DA) over efficiency (under SI-CC) entails a very large efficiency cost for teachers in our context.

Concerning stability measures, reported in Panel D of Table 3, SI-DA is, by construction SI stable contrary to SI-CC. The latter leads to 8,138 teachers being involved in at least one blocking pair not authorized in the definition of SI stability that we introduced in Section 4. If one considers all the possible blocking pairs, remember that neither SI-DA nor SI-CC are GS stable. SI-CC has 8,202 teachers involved in at least one blocking pair compared to 8,436 teachers under SI-DA. This reduction can be explained by the important efficiency gains that SI-CC has compared to SI-DA. Since much more tenured teachers move under SI-CC, the number of blocking pairs caused by teachers staying at their initial position decreases. The small differences between the three stability notions under SI-CC mean that the vast majority of blocking pairs are caused by less preferred teachers being assigned to a new region despite more preferred teachers requesting that region. For SI-DA, only 2,070 teachers are blocking due to this last reason while the remaining 6,366 block because of a tenured teacher staying at his initial region.

Fact 1 *Despite a slightly better distributional performance and a SI stable assignment, SI-DA has a tremendous mobility cost compared to SI-CC. Only 7 tenured teachers move from their initial position under SI-DA, compared to 367 under SI-CC. Imposing the SI constraint to DA-based mechanisms comes at a large efficiency cost in terms of mobility.*

6.6 Benefits and costs of distributional constraints

We now turn to a discussion of the benefits and costs of adding distributional constraints to assignment mechanisms. To do so, we compare allocations under SI-CC and under “TTC*”, the benchmark mechanism which uses the same priorities as SI-CC (incorporating teacher types) but is not status-quo improving. We also compare allocations under SI-DA and under “DA*”, although we devote less time to this comparison due to the relatively poor performance of SI-DA identified in the previous section.

Better distribution of teacher experience. Figure 1 shows the cumulative distribution of teachers’ experience in the three youngest regions of France.⁴⁶ Every year a very large number of teachers with a few years of experience leave Créteil, Versailles, and Amiens. They are replaced by an equally large number of inexperienced teachers. That structural imbalance means that the status-quo improvement requirement is unlikely to be respected by mechanisms such as DA* or TTC*. Figure 1 confirms this.

Fact 2 *In the three youngest regions of France, the SI-CC mechanism produces a distribution of teachers’ experience which first-order stochastically dominates the distribution under TTC*. SI-CC*

⁴⁶See Figure A.1 for the types distribution in regions.

assigns only 1,371 teachers with one or two years of experience to the three youngest regions, while TTC assigns 1,844 of them to these three regions. On the contrary, SI-DA produces a distribution of teachers' experience which does not first-order stochastically dominates the distribution under DA*.*

The distributions under the benchmark mechanisms need not dominate the initial distribution as they do not impose status-quo improvement. Indeed, the cumulative distribution of teachers experience under DA* and TTC* do not dominate the initial distribution.^{47,48}

Interestingly, the distributional benefits of status-quo improvement we find for SI-CC do not hold for SI-DA. In the three youngest regions of France, the SI-DA mechanism produces a distribution of teachers' experience which does not first-order stochastically dominates the distribution under DA*. SI-DA also assigns more teachers with one or two years of experience (1,039) to the three youngest regions than DA* (846). This finding confirms that imposing status-quo improvement to DA-based mechanisms can backfire. In general, one may expect that an increase in mobility upon a status-quo improving assignment can only be done at the expense of the distribution of teachers' experience (this is indeed what we observe when comparing SI-CC and TTC*). However, when mobility is extremely low, as under SI-DA, this trade-off may disappear. In essence, SI-DA just assigns new teachers to vacant positions and leaves all tenured teachers at their initial positions (only 7 of them move from their initial positions). The improvement upon the initial distribution in terms of teachers' distributions is thus minimal among tenured teachers and further movement may help improving these distributions in many regions. Indeed, even though DA* does not impose status-quo improvement, the higher mobility it creates improves the distribution of teachers' experience in these three youngest regions in France. Finally, note that the two DA-based mechanisms assign fewer inexperienced teachers to Créteil, Versailles, and Amiens than the SI-CC-based mechanisms (1,039 for SI-DA and 846 for DA*). Again, this is due to the severe lack of outgoing mobility from these regions.

To complement the results for the three youngest regions of France, we also report the results for the three oldest regions of France. The objective is now to assign younger teachers.

Fact 3 *In the three oldest regions of France, the distribution under SI-CC stochastically dominates the one under TTC* (Figure 1). SI-CC assigns 191 teachers with one or two years of experience to these regions, while SI-CC only assigns 96 of them. On the contrary, DA* produces a distribution of teachers' experience which first-order stochastically dominates the distribution under SI-DA.*

Two channels, that work in opposite directions, could a priori explain SI-CC's better performance compared to TTC*. On one hand, for tenured teachers, SI-CC prevents more teachers from moving

⁴⁷Figure A.4 shows that the distributional performance of TTC* depends on the ordering used to start chains. When the teachers starting a chain are selected randomly or by decreasing order of their maximal priority points, the distribution of teacher experience under the benchmark SI-CC mechanism does not dominates the initial distribution. It does when the teachers are selected by increasing order of their maximal priority points.

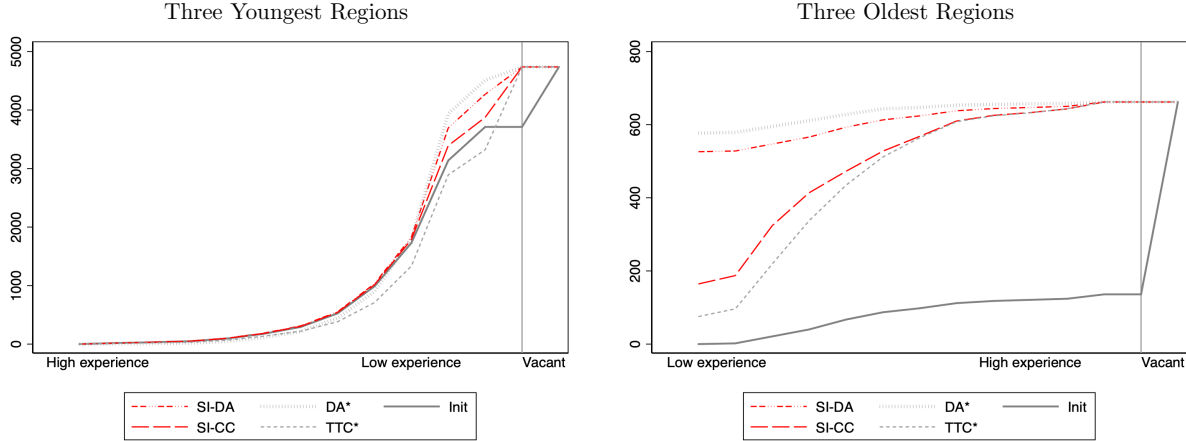
⁴⁸Appendix Figures A.2 and A.3 show that these results persist for TTC* when we consider the entire group of regions whose average experience is below the median.

away from Créteil, which limits the possibility of assigning these (relatively inexperienced) teachers to attractive regions. On the other hand, we might expect SI-CC to prevent new teachers from replacing tenured teachers in Créteil (due to new teachers' low experience) which would mechanically redirect these new teachers to attractive regions. Our results confirm that the second channel dominates the first one in terms of magnitude. Indeed, by preventing tenured teachers to move away from Créteil to attractive regions, SI-CC lowers competition for vacant seats in these attractive regions. New teachers, who prefer these regions to Créteil, are able to get assigned to these vacant seats. (As we discuss below, the distribution of ranks of new teachers is more favorable under SI-CC rather than under TTC*.)

A salient fact emerges when comparing distributional performances in the youngest and oldest regions: In the old regions, all mechanisms easily produce a distribution of teacher experience that dominates the initial distribution, while in young regions only mechanisms that respect status-quo improvement do. This finding reflects the very different levels of attractiveness of these regions. Old regions receive numerous applications from teachers, which makes it easier to improve the experience distribution. This is much more difficult in young regions that receive very few applications, especially from experienced teachers. Recall that the ratio of incoming over exiting requests is equal to 15 in Rennes but 0.03 in Créteil. This large difference in number of applications also explains why the cumulative distributions of teachers experience are very compressed in young regions, but not in old ones. Due to the limited room for improvement in disadvantaged regions, most mechanisms have a similar capped performance. Last, the performance of DA* is very good for the oldest regions since it produces a distribution of teachers' experience which dominates those of all the other mechanisms. For these regions, the mechanism gives priority to youngest teachers among those applying. Since these regions are over-demanded, the regions accepts the youngest teachers and status-quo improvement for these regions is fulfilled. Note that, under SI-DA, many young (tenured) teachers cannot apply to these regions since they are stuck in the youngest regions such as Créteil and so real demand for these oldest regions under SI-DA is lower. This high performance of DA* in the oldest regions is achieved by accepting tenured teachers from other regions at the expense of these other regions. As we can see in Figure 1, DA* violates status-quo improvement in the three youngest regions.

Lower inequalities between regions. By ensuring stochastic dominance of teachers types, the status-quo improvement condition makes sure that regions are not harmed by the reallocation of teachers. Old regions become younger, and young regions become older, hence reducing the initial differences in teachers experience between regions. While the previous paragraph discussed the distributional performance of the mechanisms for the three least and most experienced regions, we now consider performance across all regions. Figure 2 plots, for each region, the change in tenured teacher experience (proxied by the average type) between SI-CC and the initial allocation (top left figure) and between SI-DA and the initial allocation (bottom left figure). Regions are ordered, along the x-axis, by average experience of their teachers at the initial allocation, so that all regions on the left are inexperienced regions that need to receive more experienced teachers.

Figure 1: CUMULATIVE DISTRIBUTION OF TEACHER EXPERIENCE



Notes: This Figure shows the cumulative distribution of teachers experience. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Bordeaux, Rennes, and Lyon). The vertical axis reports the 12 experience bins of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The thirteenth bin corresponds to the vacant positions (the empty type). The mechanisms that are status-quo improving are plotted in red. Those that do not are in grey. The thick black line corresponds to the cumulative distribution of teachers' types at the initial assignment.

The left panel of Figure 2 shows that, compared to the initial allocation, SI-CC increases the average experience of tenured teachers in the young regions and reduces the average experience of tenured teachers in the relatively older regions. By reducing the experience gap between young and old regions, SI-CC effectively lowers existing inequalities between regions. The right panel of Figure 2 shows that SI-CC does not only reduce inequalities compared to the initial allocation, it also reduces the experience gap compared to TTC*.⁴⁹ As discussed when comparing SI-DA and its benchmark DA*, the same may not hold for DA-based mechanisms. Indeed, the extra mobility created by relaxing the status-quo constraints may generate improvements in the distributions of teachers' experience. In Figure A.5 we observe that SI-DA does not reduce the experience gap between young and old regions compared to DA*.

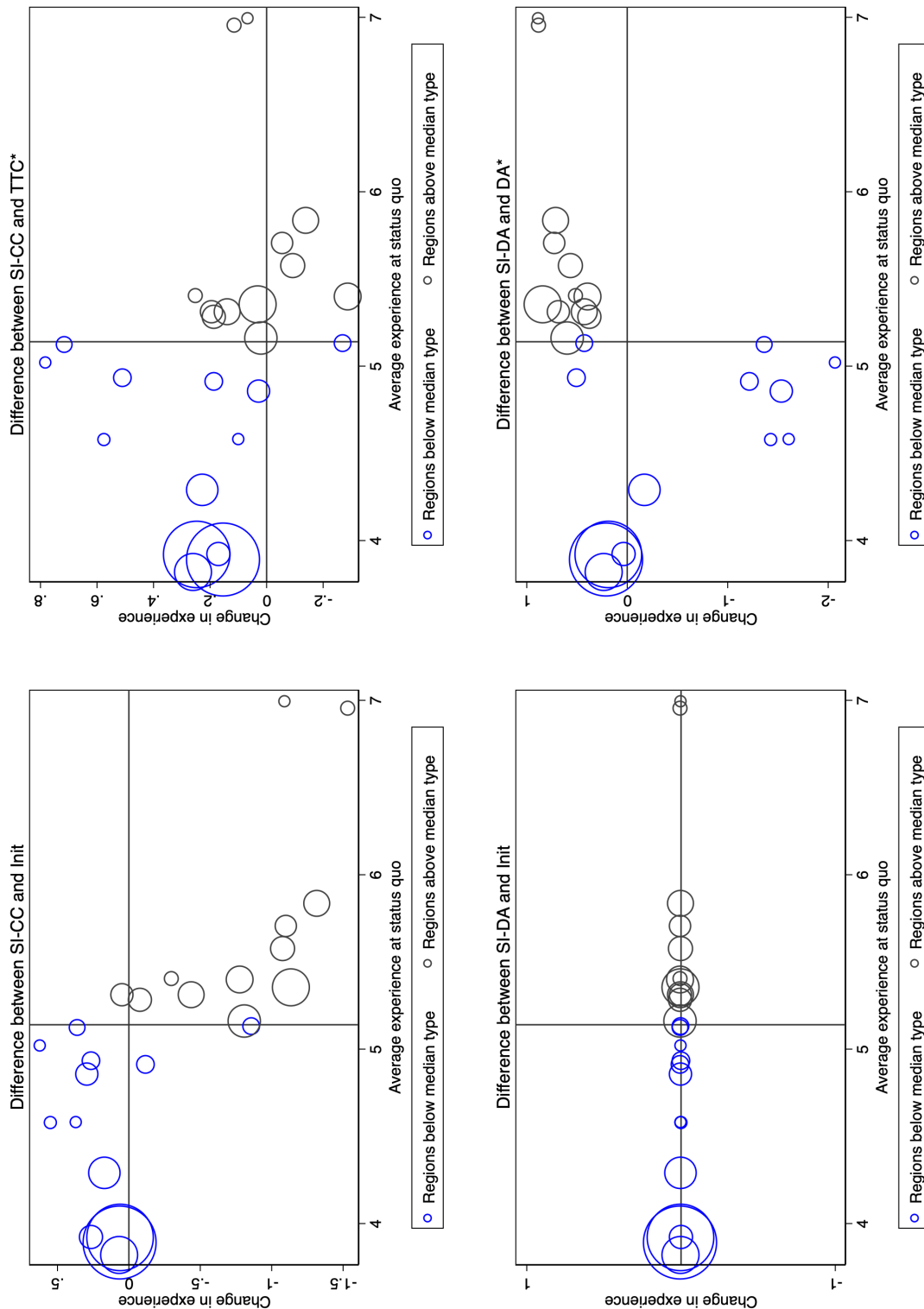
In contrast, the bottom right panel of Figure 2 brings additional evidence on the relatively poor distributional performance of SI-DA, compared to its benchmark mechanism that is not status-quo improving. SI-DA fails to reduce the average experience of teachers in the old regions compared to DA*.

Fact 4 *SI-CC reduces the large gap in teachers experience that exists at the initial allocation be-*

⁴⁹We reach similar (if not better) conclusions regarding SI-CC's better performance than its benchmark TTC* when considering not only tenured teachers but also new ones (See Figure A.5). Note, however, that including new teachers in the experience distribution complicates slightly the comparison between the initial allocation and any other allocation. This is because, in each region, the initial experience distribution can only be computed on tenured teachers— as new teachers are not assigned any region before the matching starts. The difference in the sample of teachers considered—tenured only for the initial allocation versus tenured plus new teachers for any other allocation—yields a mechanical drop in teachers average experience when comparing the final allocation under SI-CC (for instance) to the initial allocation.

tween young disadvantaged regions and older regions. *SI-CC* also reduces the gap compared to its benchmark mechanism *TTC**. In contrast, *SI-DA* slightly reduces the gap that exists at the initial allocation, *DA** further reduces the gap.

Figure 2: CHANGE IN TENURED TEACHER EXPERIENCE (TYPE) ACROSS REGIONS



Notes: This Figure shows the difference in the average experience of teachers between the allocation obtained with SI-CC against the initial allocation (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the allocation obtained with SI-DA against the initial allocation (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the x-axis) by average experience of their teachers at status quo (the initial allocation). The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than the one of the benchmark allocation to which it is compared. The name of the three least experienced regions (Cr eteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

Limited trade-off between teacher distribution and teacher welfare. Next, we investigate whether the better distributional performance of SI-CC comes at the cost of a poorer welfare for teachers, as measured by the number of teachers who obtain a new region and the rank of the region teachers obtain. Table 3 shows that the number of teachers who obtain a new assignment is larger under the benchmark mechanisms that are not status-quo improving their counterparts (SI-CC and SI-DA). Under SI-CC 1,598 tenured teachers move again 2,470 under TTC*, a difference of 872 teachers. For DA-based mechanisms, the difference is much higher since 1,260 additional tenured teachers move under DA* compared to SI-DA.

The cost in terms of teacher mobility is larger in the three youngest region (Créteil, Versailles, and Amiens, CVA for short) than in the three oldest regions (Rennes, Bordeaux, and Lyon). In CVA, satisfied mobility requests are significantly larger under TTC* (1,018) than under SI-CC (367). A low mobility for teachers in young and unattractive regions is not surprising as the status-quo improvement condition is imposing relatively more constraints for these regions—that receive very few applications to enter—than for older attractive regions. In the former, fewer teachers manage to leave with status-quo improving mechanisms as there are very few candidates that have a large enough experience to replace them.

Differences between tenured and new teachers. The differences we observe between attractive and unattractive regions might explain an interesting finding: despite a larger movement under TTC*, the rank distribution of the region that new teachers obtain under this mechanism is dominated by the distribution under SI-CC (see Panel C of Table 3). It confirms our prior explanations when comparing the mechanisms with their respective benchmarks.

Fact 5 *The distribution of ranks that tenured teachers obtain under the benchmark mechanisms that are not status-quo improving dominate the ones under SI-CC. The opposite holds for new teachers.*

On average, new teachers are assigned their 8.4th rank under SI-CC and their 10.2th rank under TTC*. This is because a much larger number of tenured teachers leave the young regions of Amiens, Créteil and Versailles under TTC* (1,018) than under SI-CC (367). These exiting teachers have to be replaced, and new teachers are the most likely substitutes. This is because very few tenured teachers ask to enter young regions and because, under TTC*, new teachers can replace tenured teachers in young regions, even if they have a lower experience. In practice, we see that 1,415 new teachers are assigned to Amiens, Créteil, or Versailles under TTC* versus 970 under SI-CC. The large share of new teachers being assigned unattractive regions under TTC* explains why these teachers are assigned lower ranked regions than under SI-CC.⁵⁰

Effects on stability measures. Last, we investigate whether imposing the status-quo improvement requirement has an impact on the number of blocking pairs. For DA-typed mechanisms,

⁵⁰Our preference estimates reveal that new teachers dislike unattractive regions less than tenured teachers. Yet, only 11.9% of the math teachers rank Créteil or Versailles as their first choice (and 14.3% of French teachers). That mild preference for unattractive regions is not large enough to justify that assigning a large share of the new teachers to these two regions will improve the ranking of the school they obtain.

imposing the status-quo improvement requirement can only create more blocking pairs since it forbids certain teachers to move from their initial position and gives a top priority to new teachers over empty slots. This is indeed what we observe since SI-DA has 442 more teachers involved in at least one blocking pair (in the sense of GS stability) compared to its benchmark. The latter being teacher-SI stable, the only blocking pairs it has are caused by teachers staying at their initial position. In addition, SI-DA has 2,070 teachers blocking because of an empty slot.

For SI-CC, the opposite happens. Indeed, since the latter and its benchmark do not impose any stability condition, the additional mobility created by relaxing the status-quo improvement constraint is done at the further expense of stability. However, this increase is limited since TTC* has only 251 additional teachers involved in a blocking pair compared to SI-CC. The small differences between the three stability notions show that both mechanisms blocking pairs are mainly driven by their high mobility rates which imply that less preferred teachers are assigned to a new region at the expense of more preferred ones who also requested that region.

Table 3: TEACHER WELFARE & TEACHER BLOCKING

	Suggested mechanisms			Benchmark mechanisms		
				Relaxing SI		
	SI-CC	SI-DA	TTC*	DA*	TTC*weakSI	DA*current
(1)	(2)	(3)	(4)	(5)	(6)	
Panel A. Teacher mobility						
Mobility - Total	5510	3919	6381	5179	6395	5863
Mobility - From the 3 youngest regions	367	1	1018	572	1058	989
Mobility - From the 3 oldest regions	116	0	117	54	116	72
Mobility - Tenured	1598	6.6	2470	1267	2484	1952
Mobility - New	3912	3912	3912	3912	3912	3912
Number of unassigned teachers	715	715	715	715	715	715
Panel B. Cumulative distribution of ranks - Tenured						
School ranked 1	773	250	1112	500	1141	833
School ranked 2	1650	919	2203	1288	2239	1775
School ranked 3	2134	1352	2758	1784	2775	2300
School ranked 4	2544	1746	3168	2224	3532	2737
School ranked higher than 5	5833	5833	5833	5833	5833	5833
Panel C. Cumulative distribution of ranks - New						
School ranked 1	1199	967	645	803	713	621
School ranked 2	1715	1557	1010	1299	1138	990
School ranked 3	2053	1942	1289	1634	1448	1263
School ranked 4	2324	2242	1535	1898	1945	1502
School ranked higher than 5	4627	4627	4627	4627	4627	4627
Panel C. Average rank of region obtained						
Average rank	7.3	8.5	7.5	8.1	7.3	8.3
Average rank - Teachers from the 3 youngest regions	7.0	7.5	5.8	6.6	5.7	5.8
Average rank - Teachers from the 3 oldest regions	2.2	9.2	2.0	6.4	2.3	5.0
Average rank - Tenured	6.5	8.5	5.3	7.0	5.3	6.0
Average rank - New	8.4	8.6	10.2	9.6	9.8	11.1
Panel D. Number of teachers blocking under the different stability notions						
GS-stability	8201	8436	8452	8010	8241	8688
Teacher-SI stability	8200	2070	8451	0	8221	8655
SI stability	8138	0	8407	0	8200	8626

† Notes: The upper part of this table reports statistics on teacher mobility. The middle part presents the cumulative distribution of the number of teachers who obtain school rank k. The bottom part of the table reports statistics on the average rank of the school teachers obtain.

We end this empirical section by discussing the efficiency-equity trade-off and the respective role played by teachers preferences and mechanisms in it. Our estimations of teachers preferences reveal that, unlike tenured teachers who dislike unattractive regions a lot, new teachers tend to have a preference for these regions. This raises a central question: Is the unbalanced distribution of teachers between regions primarily driven by teachers preferences or by the mechanism used? If preferences explain the distribution, would there still be room for 2-sided improvements by changing mechanism? For instance, in the extreme situation where all new teachers prefer Créteil and all tenured teachers prefer the attractive regions of Toulouse and Bordeaux, any equity-improving reassignment would hurt some teachers, bringing back the traditional trade-off between efficiency and equality. Our results show a different picture: our mechanisms can simultaneously improve on three dimensions: the distribution of teachers between regions, the welfare of teachers and the welfare of regions.

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Appendix A Omitted Result and Proofs

Proposition 6 *Let ψ be a status-quo improving and Pareto efficient mechanism which selects a Pareto efficient matching other than SI teacher optimal whenever such a matching exists. Then, ψ is not strategy-proof.*

Proof: On the contrary suppose ψ is strategy-proof. Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \triangleright_{s_1} \tau(t_2) \triangleright_{s_1} \tau(t_1) \triangleright_{s_1} \theta_\emptyset$, $\tau(t_1) \triangleright_{s_2} \tau(t_3) \triangleright_{s_2} \tau(t_2) \triangleright_{s_2} \theta_\emptyset$ and $\tau(t_2) \triangleright_{s_3} \tau(t_1) \triangleright_{s_2} \tau(t_3) \triangleright_{s_3} \theta_\emptyset$, $s_3 P_{t_1} s_2 P_{t_1} s_1 P_{t_1} \emptyset$, $s_1 P_{t_2} s_3 P_{t_2} s_2 P_{t_2} \emptyset$, and $s_2 P_{t_3} s_1 P_{t_3} s_3 P_{t_3} \emptyset$.

There exists a unique SI teacher optimal matching, denoted by ν , in which each teacher is assigned to her top choice. In any other status-quo improving Pareto efficient matching at most one teacher is assigned to her top choice and at least one teacher is assigned to her second choice. Suppose ψ selects Pareto efficient and status-quo improving matching μ in which t_1 is assigned to her second choice s_2 . If t_1 reports only s_3 and s_1 acceptable, then ν is the unique SI teacher optimal matching and ψ assigns t_1 to s_1 , t_2 to s_3 and t_3 to s_2 . In this updated market, if t_2 reports only s_1 and s_2 acceptable, then ν is the unique SI teacher optimal matching and ψ assigns t_1 to s_3 , t_2 to s_2 and t_3 to s_1 . Finally, in this updated market, if t_3 reports only s_2 and s_3 acceptable, then ν is the Pareto efficient and status-quo improving matching and ψ needs to select it. However, this contradicts with strategy-proofness of ψ , i.e., it is manipulated by t_3 .

We can show the same result for any Pareto efficient and status-quo improving matching in which at least one teacher is assigned to her second choice under the original market. ■

Proof of Proposition 1: On the contrary, suppose μ is SI teacher optimal and it is Pareto dominated by ν . Since μ status-quo improves ω so does ν . Hence, ν is status-quo improving. In addition, since ν Pareto-dominates μ , all teachers weakly prefer ν to μ . Because, $\nu \neq \mu$ and teachers' preferences are strict, some teachers strictly prefers ν to μ . However, this violates SI teacher optimality of μ . This is a contradiction. ■

Proof of Theorem 1: SI teacher optimality: Recall that the requirement of status-quo improvement is embedded in the definition of SI teacher optimality. Consider an arbitrary market P . Let $\hat{\mu}$ be the outcome of SI-CC under this market. We proceed in two parts.

1. We first show that $\hat{\mu}$ is status-quo improving.

First, consider teachers. Under SI-CC, each school s points to all teachers in ω_s one by one. When a teacher $t \in \omega_s$ is pointed by s in some Step k , then $s \in A_t^k$ and she can always form a one-school cycle (s, t) whenever she points to s . Similarly, any new teacher $t \in N$ can form a cycle with \emptyset in any step of SI-CC. Hence, $\hat{\mu}_t R_t \omega_t$ for all $t \in T$.

Next, consider schools. The tail of any executed chain is a new teacher. Hence, if in some step of SI-CC, a school s is sending out a teacher then it is simultaneously acquiring another

teacher; as a result $|\hat{\mu}_s| \geq |\omega_s|$. In any step k of SI-CC, when we consider the set of remaining status-quo employees and teachers assigned in the first $k - 1$ steps, because of the positive balance requirement in Improvement Condition 1, the previous observation and the fact that a teacher cannot be assigned a school if he is unacceptable, each school s is weakly better off compared to ω_s . Hence, $\hat{\mu}_s \succsim_s \omega_s$ for all $s \in S$.

We showed that $\hat{\mu}$ is status-quo improving.

2. Before proving $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers, we first state a claim that will be used in the proof.

Claim 1: For a school s , suppose step K is the final step in which school s is assigned a teacher in an executed chain such that s is the head of this chain. Let the set of remaining status-quo employees of s at the end of step K be denoted as ω_s^K and all assigned teachers to s from the beginning of step 1 until the end of step K as μ_s^K . Let ν be a matching such that all teachers assigned in the first K steps of SI-CC are assigned to their match under SI-CC and $|\nu_s| < |\omega_s^K \cup \mu_s^K|$. Then, ν is not a status-quo improving matching.

Proof of Claim 1: Suppose teacher t is assigned to s in this chain in step K and t is pointing s under Condition 2. First observe that $\mu_s^K \subseteq \nu_s$. Also notice that, if $\omega_s^K = \emptyset$, then $|\nu_s| = |\omega_s^K \cup \mu_s^K|$. Hence, $\omega_s^K \neq \emptyset$. Since Condition 1 does not hold for school s via teacher t , there exists some type θ such that $\tau(t_s^K) \succeq_s \theta \succ_s \tau(t)$ and

$$\sum_{\theta' \succeq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step K of SI-CC.⁵¹ That is, the number of teachers with weakly better type than θ in $\mu_s^K \cup \omega_s^K$ cannot be more than what it is in ω_s . Moreover, all teachers in ω_s^K have weakly better type than θ . Hence, $|\nu_s| < |\omega_s^K \cup \mu_s^K|$ and $\mu_s^K \subseteq \nu_s$ imply that the number of teachers with weakly better type than θ in ν_s is strictly less than this number in ω_s . Therefore, ν_s is not preferred to ω_s by school s . \diamond

Next, we show that $\hat{\mu}$ cannot be Pareto dominated by another status-quo improving matching for teachers.

On the contrary, suppose there exists a status-quo improving matching ν that Pareto dominates $\hat{\mu}$ for teachers. By considering the teachers assigned in each step of SI-CC inductively, we show that such a matching cannot exist, in particular we should have $\nu = \hat{\mu}$.

We denote the set of teachers assigned in step k of SI-CC under market P with T_k and union of these sets up to step k as $\bar{T}_k \equiv \cup_{k'=1}^k T_{k'}$.

Step 1: Each teacher $t \in T_1$ is assigned in $\hat{\mu}_t$ to the best school in A_t^1 . If $\nu_t P_t \hat{\mu}_t$ for some $t \in T_1$, then $\nu_t \notin A_t^1$. Thus, for school $s \equiv \nu_t$ both improvement conditions are violated via teacher t . Since this is Step 1, the current matching satisfies $\mu = \emptyset$, and hence, the current balances

⁵¹Actually, sum of balances $\sum_{\theta' \succeq_s \theta} b_s^{\theta'}$ never becomes negative in the mechanism for any type θ , as the sum starts at zero at the beginning of Step 1, and whenever it is zero, we do not admit a teacher with a type worse than θ by sending out a teacher with a type better than θ by Improvement Condition 1.

$b_s^\theta = 0$ for all schools s and types θ . The violation of Condition 1 implies that

- if there exists a teacher $t_s^1 \in \omega_s$ that s is pointing, then it has type $\tau(t_s^1) \triangleright_s \tau(t)$: thus, t has a worse type than the worst type status-quo employees of this school; and
- if such a teacher does not exist, then $\omega_s = \emptyset$.

Thus, in either case, $\nu_s \setminus \{t\} \succsim_s \omega_s$ and $|\nu_s| > |\omega_s|$ as otherwise ν is not status-quo improving for s by FOSD preferences. The violation of Condition 2 for s via t , on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s : in this case status-quo improvement for s under ν would be violated; or
- there are no new teachers: in this case, as we showed $|\nu_s| > |\omega_s|$ implies that there exists some schools s' such that $|\omega_{s'}| > |\nu_{s'}|$; as a result in this case status-quo improvement for s under ν would be violated by FOSD preferences; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher t cannot exist with $\nu_t P_t \hat{\mu}_t$. Since $\nu_t R_t \hat{\mu}_t$ for all t then for all $t \in T_1$, $\nu_t = \hat{\mu}_t$. *Inductive assumption:* For any $k > 1$, Assume that for all $k' < k$ and $t \in T_{k'}$, $\nu_t = \hat{\mu}_t$. We show that the same holds for teachers in T_k :

Step k : Each teacher $t \in T_k$ is assigned in $\hat{\mu}_t$ to the best school in A_t^k . If $\nu_t P_t \hat{\mu}_t$ for some $t \in T_k$, then $\nu_t \notin A_t^k$. Thus, for school $s \equiv \nu_t$ both improvement conditions are violated via teacher t . Noting μ is the current matching determined until the end of step $k - 1$, the violation of Condition 1 implies that

- if there exists a teacher $t_s^k \in \omega_s$ that s is pointing, then it has type $\tau(t_s^k) \triangleright_s \tau(t)$ and there exists an intermediate type θ such that $\tau(t_s^k) \triangleright_s \theta \triangleright_s \tau(t)$ with

$$\sum_{\theta' \triangleright_s \theta} |\mu_s^{\theta'}| - |\{t' \in \omega_s : \mu_{t'} \neq \emptyset\}^{\theta'}| \leq 0.$$

By the inductive assumption for the current matching $\mu_{t'} = \nu_{t'}$ for all t' assigned until this step (i.e., those in \bar{T}_{k-1}), and hence we also have

$$\sum_{\theta' \triangleright_s \theta} \left| (\nu_s \cap \bar{T}_{k-1})^{\theta'} \right| - \left| (\omega_s \cap \bar{T}_{k-1})^{\theta'} \right| \leq 0.$$

Teacher t has a worse type than the remaining worst-type status-quo employee of this school i.e., those in $\omega_s \setminus \bar{T}_{k-1}$. Thus, in ν replacing any of these employees with t would violate status-quo improvement for s in ν , as this would have led to an FOSD violation for type θ :

$$\sum_{\theta' \triangleright_s \theta} \left| \nu_s^{\theta'} \right| - \left| \omega_s^{\theta'} \right| < 0.$$

Then t does not replace any of the remaining status-quo employees, but she is an additional

teacher acquired: $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

- if such a teacher does not exist, then $\omega_s \setminus \bar{T}_{k-1} = \emptyset$, and hence, as $t \in \nu_s \setminus \bar{T}_{k-1}$ we have $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$.

Observe that in the algorithm at each step we make sure that each school acquires at least as many teachers as it sends out and hence, for the current matching $|\mu_s| \geq |\omega_s \cap \bar{T}_{k-1}|$. Since $\mu_s = \nu_s \cap \bar{T}_{k-1}$ by the inductive assumption, we have $|\nu_s \cap \bar{T}_{k-1}| \geq |\omega_s \cap \bar{T}_{k-1}|$. Therefore, as we also showed that $|\nu_s \setminus \bar{T}_{k-1}| > |\omega_s \setminus \bar{T}_{k-1}|$ above, we obtain $|\nu_s| > |\omega_s|$.

The violation of Condition 2 for s via t , on the other hand, implies one of the following conditions to hold:

- t is not acceptable for s : in this case status-quo improvement for s under ν would be violated; or
- there are no remaining new teachers: Claim 1 implies that there exists at least one school s' such that $\nu_{s'}$ does not status-quo improve upon $\omega_{s'}$; or
- $q_s = |\omega_s|$: in this case, as we showed $|\nu_s| > |\omega_s|$, $|\nu_s| > q_s$ contradicting the feasibility of ν as matching.

Then, Condition 2 cannot be violated as none of these conditions hold, which is a contradiction. Hence, such a teacher $t \in T_k$ with $\nu_t P_t \hat{\mu}_t$ cannot exist.

Since $\nu_t R_t \hat{\mu}_t$ for all t then for all $t \in T_k$, $\nu_t = \hat{\mu}_t$, completing the induction and showing that $\nu = \hat{\mu}$.

Strategy-proofness: We state two claims that we will use in the proof.

Claim 2: Suppose teacher t is assigned in step K of SI-CC. For any $k < K$, then $A_t^{k+1} \subseteq A_t^k$.

Proof of Claim 2: Let $s \notin A_t^k$. We will show that $s \notin A_t^{k+1}$. We consider two possible cases.

Case 1: s does not have an unfilled seat at step k : First notice that, if there is no remaining status-quo employee of s in step k , then it should have been removed in an earlier step of SI-CC. Then, there exists some type θ such that $\tau(t_s^k) \succeq_s \theta \succ_s \tau(t)$ with

$$\sum_{\theta' \succeq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If school s is part of the executed cycle or chain in step k , then the teacher assigned to s has a type weakly better than type θ under \triangleright_s and similarly, the teacher leaving school s , namely, t_s^k also has a type weakly better than type θ . Hence, after executing the cycle in step k relation above still holds. Moreover, s cannot send out a status-quo employee without getting a new one by the definition of SI-CC. Similarly, s cannot get a teacher without sending a status-quo employee. If school s is not part of the executed cycle or chain in step k , equation above still holds. In either case, $s \notin A_t^{k+1}$.

Case 2: s has an unfilled seat at step k : Either t is unacceptable for s or t is acceptable for s but there does not exist a remaining new teacher in step k . If the former case holds, then $s \notin A_t^{k+1}$ by definition.

If the latter case holds, then either s does not have remaining status-quo employee or there exists some type θ such that $\tau(t_s^k) \succeq_s \theta \succ_s \tau(t)$ and

$$\sum_{\theta' \succeq_s \theta} b_s^{\theta'} \leq 0$$

where $b_s^{\theta'}$ is the current balance of type θ' at step k of SI-CC. If the former subcase holds, then neither Condition 1 nor Condition 2 holds for t in step $k + 1$. For the later condition, we refer to Case 1 above. Hence, $s \notin A_t^{k+1}$. \diamond

Claim 3: Consider a step k of SI-CC mechanism such that there exists a path of schools and teachers $(s_1, t_1, s_2, t_2, \dots, s_\ell, t_\ell)$ in which school $s_{\ell'}$ points to teacher $t_{\ell'}$ and teacher $t_{\ell'-1}$ points to school $s_{\ell'}$ for each $\ell' < \ell$ and $s_1 \in A_{t_\ell}^k$. If none of the schools in this path are assigned a teacher in this step, the same path forms in step $k + 1$ and $s_1 \in A_{t_\ell}^{k+1}$.

Proof of Claim 3: As no teacher is assigned to the schools of the path in step k , the teachers in the path remain in the step $k + 1$. Since $t_{\ell'} \equiv t_{s_{\ell'}}^k$ is the highest priority remaining status-quo employee in step k of school $s_{\ell'}$, she continues to be so in step $k + 1$, thus, school $s_{\ell'}$ points to $t_{\ell'}$ in step $k + 1$. No other status quo employee of these schools is assigned to any other school in step k , either, because the assignment of status-quo employees requires the school pointing to them and each school points to at most one teacher in this step. Thus, as Condition 1 or Condition 2 holds for each school $s_{\ell'}$ via teacher $t_{\ell'-1}$ (in modulo ℓ , thus $t_0 \equiv t_\ell$) in step k , the same condition continues to hold in step $k + 1$ via the same teacher. Hence, $s_{\ell'} \in A_{t_{\ell'-1}}^{k+1}$ for each ℓ' . Since $A_{t_{\ell'-1}}^{k+1} \subseteq A_{t_{\ell'-1}}^k$ by Claim 2, and $s_{\ell'}$ is the favorite school of teacher $t_{\ell'-1}$ in the opportunity set in step k , we still have $s_{\ell'}$ as the favorite school of teacher $t_{\ell'-1}$ in step $k + 1$ and she continues to point to $s_{\ell'}$ in Step $k + 1$. \diamond

We are ready to finish the proof for the strategy-proofness of SI-CC. Recall that we denote the set of teachers assigned in step k of SI-CC with T_k . First, notice that a teacher t' cannot change the schools in $A_{t'}^1$ by misreporting her preferences since $A_{t'}^1$ does not depend on the submitted preferences. Moreover, by Claim 2, $\{A_t^k\}$, the opportunity sets for teacher t , weakly shrink in through out SI-CC. Hence, a teacher t cannot be assigned to a school $s \notin A_t^1$ under SI-CC. We first consider the teachers in T_1 . Each $t \in T_1$ is assigned to her best choice in A_t^1 . Hence, any teacher $t \in T_1$ cannot benefit from misreporting her preferences.

Next, we consider a teacher $t \in T_2$. As explained above, teacher t cannot be assigned to school $s \notin A_t^1$ under SI-CC. Teacher $t \in T_2$ is assigned to best school in A_t^2 when she submits her true preferences. We denote the best school in A_t^2 according to P_t with s' . By Claim 2, $A_t^2 \subseteq A_t^1$. Hence, if $t \in T_2$ can benefit from misreporting her preferences, then she is assigned to some school $s \in A_t^1 \setminus A_t^2$. If $A_t^1 \equiv A_t^2$, then t cannot benefit from misreporting her preferences. Suppose $A_t^1 \setminus A_t^2 \neq \emptyset$. We will show that t cannot be assigned to a school $s \in A_t^1 \setminus A_t^2$ such that $s P_t s'$ by misreporting. Particularly, we show t cannot prevent the cycle or chain executed in step 1 without hurting herself.

First notice that, if t forms a cycle in step 1 by misreporting and pointing to some school $s'' \in A_t^1$, then by Claim 3, $s'' \in A_t^2$ and the path leading to t in this cycle starting with school s'' forms again when she submits P_t , which does not match her in step 1. Hence, any such school s'' cannot be preferred to s' , i.e., t 's assignment under truth-telling.

If a chain is executed in step 1, teacher t cannot be a part of that chain by misreporting and pointing some other school in A_t^1 . This follows from the fact that the executed chain starts with a specific new teacher and a teacher \bar{t} , who is pointed by her status-quo school \bar{s} , can only be added to the executed chain if a previously included teacher points to \bar{s} , independent of \bar{t} 's preference. Teacher t can prevent the executed chain by only forming a cycle by misreporting. However, as explained above, under such a cycle t will be assigned to a school weakly worse than s' .

Moreover, with a similar reasoning to a chain, teacher t cannot affect the executed cycles in step 1 by submitting a different preference without being matched in step 1 in a new cycle (and therefore, making her weakly worse off as we showed above).

By using similar arguments, we can show that any teacher in T_k where $k > 2$ cannot benefit from misreporting her preferences. ■

Proof of Proposition 2: We first show the existence of Gale-Shapley stable matching. Consider a market P . We construct a strict rank order list, $\hat{\succ}_s$, for each school s over the teachers as follows: for any $t, t' \in T$

- if $\tau(t) \triangleright_s \tau(t')$ then $t \hat{\succ}_s t'$;
- if $\tau(t) = \tau(t')$, then the relative order between t and t' is determined arbitrarily;
- $\tau(t) \triangleright_s \theta_\emptyset$ if and only if $t \hat{\succ}_s \emptyset$.

It is easy to verify that the outcome of teacher-proposing DA mechanism (Abdulkadiroğlu and Sönmez, 2003b) under $(P, \hat{\succ})$ is Gale-Shapley stable.

Next, we show that for some market there does not exist a Gale-Shapley stable and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\}, \omega_{s'} = \{t_2\},$$

with quotas $q_s = q_{s'} = 1$, type rankings $\tau(t_1) \triangleright_s \tau(t_2) \triangleright_s \theta_\emptyset$, and $\tau(t_1) \triangleright_{s'} \tau(t_2) \triangleright_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s' P_{t_1} s P_{t_1} \emptyset,$$

$$s' P_{t_2} s P_{t_2} \emptyset,$$

Under this market, unique status-quo improving matching is ω . However, ω is blocked by (t_1, s') . ■

Proof of Proposition 3: Under the current French mechanism, when there are no empty seats, each school fills its capacity and only the status-quo employees are assigned to the schools.

This follows from the fact that teachers in ω_s have the q_s highest priority at school s and they are considering their status-quo school acceptable. Hence, if there is a blocking pair (t, s) and $\tau(t) \succ_s \tau(t')$ and t' is assigned to s , then $t' \in \omega_s$. Teacher-SI stability and status-quo improving property of ω directly follows from the definition.

Next, by slightly modifying the example in the proof of Proposition 2, we show that when there are no empty seats at schools the current French mechanism is not status-quo improving. Consider the example in the proof of Proposition 2 such that teacher t_2 prefers school s most. Then, the French mechanism assigns t_1 and t_2 to s' and s , respectively. This matching is not status-quo improving for school s .

Finally, via an example, we show that when there are empty seats at some school, then there does not exist a teacher-SI stable and status-quo improving matching. Let $S = \{s, s'\}$, $T = \{t_1\}$, $\omega_s = \{t_1\}$, $\omega_{s'} = \emptyset$. Each school has one available seat and t_1 prefers s' to s . The unique status-quo improving matching is ω but it is blocked by (t_1, s') . Hence, it is not teacher-SI stable. \blacksquare

Proof of Proposition 4: Substitutes: On the contrary, we suppose there exist $\bar{T} \subseteq T$ and distinct $t, t' \in \bar{T}$ such that $t \in C_s(\bar{T})$ and $t \notin C_s(\bar{T} \setminus \{t'\})$. There exists some other teacher t'' who was assigned to s^k under $C_s(\bar{T} \setminus \{t'\})$, where s^k is t 's slot under $C_s(\bar{T})$. Consider the execution of the algorithm to determine $C_s(\bar{T} \setminus \{t'\})$ in step k when t'' is assigned to s^k : as t is not assigned in $C_s(\bar{T} \setminus \{t'\})$, she is still available and is not picked by slot s^k ; thus, $t'' \succ_s^k t$. As a consequence, when the algorithm was executed to determine $C_s(\bar{T})$, teacher t'' was already assigned to a slot $s^{k''}$ such that $k'' < k$ so that she was not available when t was assigned s^k .

We will show that such a teacher t'' cannot exist, leading to a contradiction and completing the proof for the substitutes condition.

Claim: There is no teacher \bar{t} such that she is assigned to a slot $s^{\hat{k}}$ in $C_s(\bar{T} \setminus \{t'\})$ and to a slot $s^{\bar{k}}$ in $C_s(\bar{T})$ such that $\bar{k} < \hat{k}$.

Proof of Claim: Suppose to the contrary such a teacher \bar{t} exists. Let \bar{t} be chosen such that \bar{k} is the smallest such index among the indexes of slots filled by such teachers in $C_s(\bar{T})$.

If slot $s^{\bar{k}}$ is unfilled in $C_s(\bar{T} \setminus \{t'\})$, then as \bar{t} is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T})$ by the supposition, we should have $\emptyset \succ_s^{\bar{k}} \bar{t}$. But then teacher \bar{t} cannot be assigned to $s^{\bar{k}}$ in $C_s(\bar{T})$.

If a teacher \hat{t} is assigned to $s^{\bar{k}}$ in $C_s(\bar{T} \setminus \{t'\})$, then as \bar{t} is still available when slot $s^{\bar{k}}$ is filled in determining $C_s(\bar{T})$ by the supposition. Therefore, by the choice of \bar{k} , teacher \hat{t} is not assigned a slot preceding $s^{\bar{k}}$ in $C_s(\bar{T})$. Therefore, she is available when $s^{\bar{k}}$ is filled in $C_s(\bar{T})$. Yet she is not picked even though $\hat{t} \succ_s^{\bar{k}} \bar{t} \succ_s^{\bar{k}} \emptyset$, a contradiction. Thus, such a teacher \bar{t} cannot exist. \diamond

Law of Aggregate Demand: On the contrary, we suppose there exists $\bar{T} \subseteq T$, $t \notin \bar{T}$ and $|C_s(\bar{T})| > |C_s(\bar{T} \cup \{t\})|$. Then, there exists a slot s^k which is filled under $C_s(\bar{T})$ but not under $C_s(\bar{T} \cup \{t\})$. However, due to the above Claim in the proof for the substitutes condition, the teacher who was assigned s^k in $C_s(\bar{T})$ is available when s^k is being filled in $C_s(\bar{T} \cup \{t\})$. Then this slot

cannot be empty in $C_s(\bar{T} \cup \{t\})$ as this teacher is acceptable for the slot, which is a contradiction. We showed that $|C_s(\bar{T})| \leq |C_s(\bar{T} \cup \{t\})|$.

By the repeated application of this argument, we conclude that whenever $\bar{T} \subseteq \hat{T}$, $|C_s(\bar{T})| \leq |C_s(\hat{T})|$. ■

Proof of Theorem 2: Strategy-proofness: It was shown by Hatfield and Milgrom (2005) that whenever the choice rules of schools satisfy the substitutes and law of aggregate demand conditions, the resulting mechanism through DA is strategy-proof for teachers. Since for each school s , auxiliary choice rule C_s satisfies these conditions and only incomplete information is about the preferences of teachers, SI-DA is strategy-proof.

SI-Stability: Suppose Assumption 1 holds. Let SI-DA outcome be μ . We will show that it is status-quo improving first. By our construction of the slot priorities, a teacher t will be accepted by ω_t whenever she applies and she will never be rejected in the further steps. Hence, it is status-quo improving for teachers. Consider the schools. First, we prove the following claim.

Claim: Each school fills all its seats in μ .

Proof of Claim: To see this, notice that no teacher t is assigned to a school s that is less preferred to ω_t in μ . Therefore, all teachers who were employed at the status quo are assigned to some school in μ . Moreover, we claim that exactly $\sum_{s \in \mathcal{S}} (q_s - |\omega_s|)$ new teachers are assigned in μ . On the contrary, suppose this claim does not hold. Then, at least one seat of a school s is unfilled in μ and this matching leaves at least one new teacher $t \in N$ unmatched such that she considers all schools with empty seat acceptable and is acceptable at all schools with empty seats at status quo. In determining $C_s(B_s^{K+1})$, where K is the final step of the DA algorithm, if the slot corresponding to this empty seat is one of slots s^k that was unfilled at the status quo, then an unassigned new teacher would have applied to that school and have been assigned to that slot by Assumption 1. Thus, this slot is filled at the status quo.

Then as all employed teachers at status quo are assigned to some school in μ , there exists a teacher $\hat{t} \notin N$ assigned in μ to a slot \hat{s}^k that was unfilled at the status quo at some school \hat{s} .

Since new teacher t is unassigned in μ , she should have applied to all schools with empty seats at status quo (which she considers acceptable by Assumption 1) including \hat{s} . Since \hat{s} has an unfilled seat at status quo, by Assumption 1, it considers t acceptable. Moreover, at the slots that are unfilled at the status quo, acceptable new teachers have higher ranking than employed teachers at the status quo by construction of the slot rankings: $t \succ_{\hat{s}}^k \hat{t}$. Thus, slot \hat{s}^k should have held t instead of \hat{t} , a contradiction.

Hence, all seats are filled in μ . ◇

Since all seats are filled in μ , by our construction of the rankings of the slots, the assignment FOSD the status-quo matching ω . Hence, μ is also status-quo improving for schools.

Next, we will show that there is no blocking pair of μ that is not allowed. By construction of

slot rankings for a school s , for teachers neither in ω_s nor in N , the type ranking of the school is respected in rankings of its slots.

Suppose there exists a blocking teacher-school pair (t, s) of μ , i.e., t prefers s to μ_t and there exists a teacher $\hat{t} \in \mu(s)$ such that $\tau(t) \triangleright_s \tau(\hat{t}) \triangleright_s \theta_\emptyset$ for some $\hat{t} \in \mu(s)$, as all seats of s are filled in μ .

Then t should have made an offer to s that was rejected in the DA algorithm. Then all teachers assigned to all slots of s have higher ranking in that slot than t . If \hat{t} was assigned in μ to a filled slot at status quo then $\hat{t} \in \omega_s$ and $t \notin \omega_s$ as $\tau(t) \triangleright_s \tau(\hat{t})$. If \hat{t} was assigned in μ to an unfilled slot at status quo then $\hat{t} \in N$ and $t \notin N$ as $\tau(t) \triangleright_s \tau(\hat{t})$. By definition of SI-stability, then (t, s) is an allowed blocking pair of μ . As such a blocking pair (t, s) and such a teacher \hat{t} is arbitrary, μ is SI-stable. ■

Proof of Proposition 5: We use the following lemma in our proof.

Lemma 1 For any $\bar{T} \subseteq T$, $\hat{D}_s(\bar{T}) \subseteq D_s(\bar{T})$.

Proof: Let s^k and s^ℓ (s^ℓ and s^k) be m^{th} and $(m+1)^{\text{th}}$ seats under \blacktriangleright_s ($\hat{\blacktriangleright}_s$), respectively. Since the relative positions of the first $(m-1)$ seats are the same under \blacktriangleright_s and $\hat{\blacktriangleright}_s$, the same teachers are assigned to the first $(m-1)$ seats by D_s and \hat{D}_s . Therefore, we consider the same set of teachers for the m^{th} seat under both \blacktriangleright_s and $\hat{\blacktriangleright}_s$. Let \tilde{T} be the set of teachers considered for the m^{th} seat under both \blacktriangleright_s and $\hat{\blacktriangleright}_s$.

Recall that, by our construction, the set of teachers acceptable for seat s^ℓ is a (weak) superset of the teachers acceptable for seat s^k . Hence, if there does not exist an acceptable teacher in \tilde{T} for seat s^ℓ , then there does not exist an acceptable teacher in \tilde{T} for seat s^k . If there is no acceptable teacher in \tilde{T} for seat s^k but there is some acceptable teacher for seat s^ℓ , then that teacher is assigned to s^ℓ under both choice functions. Since the relative positions of the remaining seats are the same under \blacktriangleright_s and $\hat{\blacktriangleright}_s$, we have $D_s(\tilde{T}) = \hat{D}_s(\tilde{T})$ whenever the set of acceptable teachers in \tilde{T} for either s^k or s^ℓ is empty.

Now suppose there exist acceptable teachers in \tilde{T} for seats s^k and s^ℓ . Let t^k and t^ℓ be the highest ranked teachers for seats s^k and s^ℓ among the ones in \tilde{T} , respectively.

If $t^k \neq t^\ell$, then under both choice rules D_s and \hat{D}_s t^k and t^ℓ are assigned to seats s^k and s^ℓ , respectively. Since the relative positions of the remaining seats are the same under \blacktriangleright_s and $\hat{\blacktriangleright}_s$, we have $D_s(\tilde{T}) = \hat{D}_s(\tilde{T})$.

If $t^k = t^\ell = t'$, then t' is assigned to seats s^k and s^ℓ under choice rules D_s and \hat{D}_s , respectively. Next, we consider the teachers in $\tilde{T} \setminus t'$. First notice that, the status-quo employee teachers who are having the highest priority among all teachers in T for s^k and s^ℓ cannot be in $\tilde{T} \setminus t'$. This would conflict with the fact that t' has the highest priority for both seats among the teachers in \tilde{T} . Then, there is one teacher in $\tilde{T} \setminus t'$ who has highest priority for both s^k and s^ℓ . We denote such teacher with t'' . If t'' is acceptable for both s^k and s^ℓ , then t'' is assigned to s^ℓ and s^k under both choice rules D_s and \hat{D}_s , respectively. If t'' is unacceptable for both s^k and s^ℓ , then no teacher is assigned

to s^ℓ and s^k under both choice rules D_s and \hat{D}_s , respectively. Under both cases, since the relative positions of the remaining seats are the same under \blacktriangleright_s and $\hat{\blacktriangleright}_s$, we have $D_s(\bar{T}) = \hat{D}_s(\bar{T})$. We are left with one remaining case: t'' is acceptable for s^ℓ but not for s^k . Then, t'' is assigned to s^ℓ under D_s but s^k is not filled under \hat{D}_s . Then, when we consider the remaining seats under both choice rules and the remaining teachers, we can treat the assignment is done via DA mechanism where each teacher ranks the seats according to their positions under \blacktriangleright_s and $\hat{\blacktriangleright}_s$. Since DA is population monotonic and individually rational, any teacher assigned under \hat{D}_s is assigned under D_s . But the other way is not always true. ■

Now, consider a sequential application of DA algorithm in which we allow teachers one by one as long as they do not apply to school s (see Dur, Kominers, Pathak and Sönmez (2018) for details). Then, eventually, we will have a set of teachers \bar{T} who have been rejected from their all choices better than s . Once all teachers apply to s , Lemma 1 implies that the rejected teachers under D is a subset of the rejected teachers under \hat{D} . Then, we allow only the rejected teachers under both choice rules from s and all other teachers who have not applied to s to apply one by one. Following this procedure will give us matching μ assignment for all schools except s under both choice rules. Moreover, DA algorithm terminates under choice rule D . However, by Lemma 1, there might be teachers rejected from s and have not applied to their next best choice under \hat{D} . That is, we may observe some teachers to be rejected from their assignment under μ . Hence, no teacher t prefers $\hat{\mu}_t$ to μ_t . ■

Appendix B Examples

In Example 6, we show that, in the same setting as Combe et al. (2020), SI-CC is not equivalent to the teacher optimal selection of TO-BE they propose.⁵²

Example 6 Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2, t'_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2, t'_2\}$, $q_{s_1} = 1$ and $q_{s_2} = 2$. Let $\tau(t_1) = \theta_1$, $\tau(t_2) = \theta_2$, $\tau(t'_2) = \theta'_2$. Finally, the preferences of the schools over types are: $\theta_2 \triangleright_{s_1} \theta'_2 \triangleright_{s_1} \theta_1$ and $\theta_1 \triangleright_{s_2} \theta_2 \triangleright_{s_2} \theta'_2$. One can check that the matching returned by the teacher optimal selection of TO-BE matches⁵³ t_1 to s_1 and t_2 to s_1 while SI-CC matches t_1 to s_1 but t'_2 to s_1 .

In Example 7, we show that M-convexity of the policy goals is not sufficient anymore to ensure existence of SI teacher optimal and strategy-proof mechanism.

Example 7 Let $S = \{s_1, s_2\}$, $T = N = \{t_1, t_2\}$, $\omega_{s_1} = \omega_{s_2} = \emptyset$, $q_{s_1} = q_{s_2} = 1$ and $\tau(s_1) = \tau(s_2) = \theta$. Suppose the constraint over the distribution of teachers require that a teacher of type θ is assigned to s_1 . This is a constraint fixing a floor which is known to be M-convex. Suppose both

⁵²Combe et al. (2020) already noted that their class of TO-BE mechanisms did not entirely defined the class of statu-quo improving, strategy-proof and two-sided efficient mechanisms. However, they did not investigate it further. Our example suggests that other non-trivial mechanisms, such as SI-CC, exist outside their class.

⁵³One can easily check that this example is well defined in their setting. Just set the preferences of the schools over the teachers being equivalent to the schools' ranking over their corresponding types.

teachers rank s_2 ahead of s_1 and s_1 ahead of \emptyset . If teachers report their true preferences, then there will be a teacher assigned to s_1 under any SI teacher optimal (or Pareto efficient) matching. Then, the teacher assigned to s_1 , say t_1 , has an incentive to claim that s_1 is unacceptable to her. Indeed, any SI teacher optimal mechanism must then assign t_2 to s_1 and t_1 to s_2 .

In Example 8, we show that in some market there does not exist a SI teacher optimal and SI-stable matching.

Example 8 Let $S = \{s_1, s_2, s_3\}$, $T = \{t_1, t_2, t_3\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$, $\omega_{s_3} = \{t_3\}$ and $q_{s_1} = q_{s_2} = q_{s_3} = 1$. Let $\tau(t_3) \triangleright_{s_1} \tau(t_2) \triangleright_{s_1} \tau(t_1) \triangleright_{s_1} \theta_\emptyset$, $\tau(t_3) \triangleright_{s_2} \tau(t_1) \triangleright_{s_2} \tau(t_2) \triangleright_{s_2} \theta_\emptyset$ and $\tau(t_1) \triangleright_{s_3} \tau(t_2) \triangleright_{s_3} \tau(t_3) \triangleright_{s_3} \theta_\emptyset$, $s_2 P_{t_1} s_1 P_{t_1} s_3 P_{t_1} \emptyset$, $s_1 P_{t_2} s_2 P_{t_2} s_3 P_{t_2} \emptyset$, and $s_1 P_{t_3} s_2 P_{t_3} s_3 P_{t_3} \emptyset$.

Let $\mu_{s_1} = t_2$, $\mu_{s_2} = t_1$ and $\mu_{s_3} = t_3$. Notice that, μ is SI teacher optimal. Under this market, ω is the unique SI-stable matching and μ Pareto dominates ω for teachers.

In Examples 9 - 11, we show that if we exclude the first condition from the definition of SI-stability, then for some market there does not exist a SI-stable matching.

Example 9 Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \{t_2\}$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_2) \triangleright_s \tau(t_1) \triangleright_s \theta_\emptyset$ for both $s \in S$ and $s_1 P_t s_2 P_t \emptyset$ for all $t \in T$.

In this market, the unique status-quo improving matching is ω . However, it is blocked by (t_2, s_1) . Hence, SI-stable matching does not exist in this market when the first condition is excluded.

Next, via example we show that if we exclude the second condition from the definition of SI-stability, then for some market there does not exist a SI-stable matching.

Example 10 Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_1) \triangleright_s \tau(t_2) \triangleright_s \theta_\emptyset$ for both $s \in S$, $s_2 P_{t_1} s_1 P_{t_1} \emptyset$ and $s_2 P_{t_2} \emptyset P_{t_2} s_1$.

Under this market, in any status-quo improving matching t_1 is assigned to s_1 . However, any such matching is blocked by (t_1, s_2) . Hence, SI-stable matching does not exist in this market when the second condition is excluded.

One can wonder if there exists a strategy-proof mechanism which selects a matching which is stable when one of the conditions is excluded whenever such a matching exists and selects a stable matching under both conditions, otherwise. In the following example, we show that such a mechanism does not exist.

Example 11 Let $S = \{s_1, s_2\}$, $T = \{t_1, t_2\}$, $\omega_{s_1} = \{t_1\}$, $\omega_{s_2} = \emptyset$ and $q_{s_1} = q_{s_2} = 1$. Let $\tau(t_2) \triangleright_{s_1} \tau(t_1) \triangleright_{s_1} \theta_\emptyset$, $\tau(t_1) \triangleright_{s_2} \tau(t_2) \triangleright_{s_2} \theta_\emptyset$ for both $s \in S$, $s_2 P_{t_1} s_1 P_{t_1} \emptyset$ and $s_2 P_{t_2} s_1 P_{t_2} \emptyset$.

Under this market, there exists a unique stable matching when condition 2 is excluded: t_1 is assigned to s_2 and t_2 is assigned to s_1 . Hence, it will be selected.

Suppose teacher t_2 reports $s_2 P'_{t_2} \emptyset P'_{t_2} s_1$. Then, we have the same problem as in Example 10. Since there does not exist a stable matching where condition 2 is excluded, we consider stable matchings when condition 2 is included. There exists a unique stable matching in which t_1 is assigned to s_1 and t_2 is assigned to s_2 . Hence, t_2 is better off by manipulating.

In Examples 12-14, we relax the conditions of Assumption 1 one by one and show that existence of SI-stable outcome may not be guaranteed.

Example 12 We consider a market in which there does not exist $N' \subseteq N$ such that $|N'| \geq \sum_{s \in S} (q_s - |\omega_s|)$.

Let $S = \{s, s'\}$, $T = \{t_1\}$, the status-quo matching be

$$\omega_s = \{t_1\}, \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, and teacher t_1 is acceptable for both schools. The preferences of the teacher t_1 is

$$s' P_{t_1} s P_{t_1} \emptyset.$$

In this market, ω_s is the unique status-quo improving matching but it is blocked by (t_1, s') .

Example 13 We consider a market in which there exists $N' \subseteq N$ such that $|N'| \geq \sum_{s \in S} (q_s - |\omega_s|)$ and each teacher in N' is acceptable for all schools with excess capacity but not all teacher in N' consider all schools with excess capacity acceptable.

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\}, \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \triangleright_s \tau(t_2) \triangleright_s \theta_\emptyset$ and $\tau(t_1) \triangleright_{s'} \tau(t_2) \triangleright_{s'} \theta_\emptyset$. The preferences of the teachers are

$$s' P_{t_1} s P_{t_1} \emptyset,$$

$$s P_{t_2} \emptyset P_{t_2} s'.$$

In this market, ω is the unique status-quo improving matching but it is blocked by (t_1, s') .

Example 14 We consider a market in which there exists $N' \subseteq N$ such that $|N'| \geq \sum_{s \in S} (q_s - |\omega_s|)$ and all teacher in N' consider all schools with excess capacity acceptable but some teacher in N' is not acceptable for some school with excess capacity.

Let $S = \{s, s'\}$, $T = \{t_1, t_2\}$, the status-quo matching be

$$\omega_s = \{t_1\}, \omega_{s'} = \emptyset,$$

$q_s = q_{s'} = 1$, $\tau(t_1) \triangleright_s \tau(t_2) \triangleright_s \theta_\emptyset$ and $\tau(t_1) \triangleright_{s'} \theta_\emptyset \triangleright_{s'} \tau(t_2)$. The preferences of the teachers are

$$s' P_{t_1} s P_{t_1} \emptyset,$$

$$s P_{t_2} s' P_{t_2} \emptyset.$$

In this market, ω_s is the unique status-quo improving matching but it is blocked by (t_1, s') .

Appendix C Extensions

C.1 Responsive preferences and Impossibility

In this section, we extend the school rankings. Instead of FOSD relation, we assume schools ranking over the types of teachers are responsive.

Each school s has strict ranking over the types and no type option denoted by θ_\emptyset denoted with \triangleright_s . For school s , type θ teachers are acceptable if and only if $\theta \triangleright_s \emptyset$. Given \triangleright_s , the preference order of school s over $T \cup \{\emptyset\}$ is given as:

- $\tau(t) \triangleright_s \tau(t')$ if and only if $t \succ_s t'$;
- $\tau(t) = \tau(t')$ if and only if $t \sim_s t'$;
- $\tau(t) \triangleright_s \theta_\emptyset$ if and only if $t \succ_s \emptyset$.

For any $|\bar{T}| < q_s$ responsiveness implies that for any $t, t' \in T \setminus \bar{T}$

- $\bar{T} \cup \{t\} \succ_s \bar{T}$ if and only if $t \succ_s \emptyset$;
- $\bar{T} \cup \{t\} \succ_s \bar{T} \cup \{t'\}$ if and only if $t \succ_s t'$.

Note that, responsive preferences is more general than FOSD. In particular, if μ_s first-order stochastically dominates matching ω_s , then $\mu_s \succsim_s \omega_s$. However, the other way may not be true. We illustrate this in the following example.

Example 15 Let $\omega_s = \{t_1, t_2, t_3, t_4\}$ such that $\tau(t_1) \triangleright_s \tau(t_2) = \tau(t_3) \triangleright_s \tau(t_4)$. Consider the following matching $\mu_s = \{t_1, t'_1, t_4, t'_4\}$ such that $\tau(t_1) = \tau(t'_1) \triangleright_s \tau(t_4) = \tau(t'_4)$. Matching μ_s does not first-order stochastically dominate ω_s . However, it is possible that $\mu_s \succ_s \omega_s$.

The following example shows that, with responsive preferences, there is no mechanism that is SI teacher optimal and strategy-proof.

Example 16 There are 6 teachers, $T = \{t_1, t'_1, t_2, t'_2, t, t'\}$, and 4 schools, $S = \{s_1, s_2, s, s'\}$. Let $\omega_{s_1} = \{t_1, t'_1\}$, $\omega_{s_2} = \{t_2, t'_2\}$, $\omega_s = \{t\}$ and $\omega_{s'} = \{t'\}$. Schools s_1 and s_2 's ranking over teacher types are:

$$\begin{aligned} \tau(t) \triangleright_{s_1} \tau(t_1) \triangleright_{s_1} \tau(t'_1) \triangleright_{s_1} \tau(t') \\ \tau(t') \triangleright_{s_2} \tau(t_2) \triangleright_{s_2} \tau(t'_2) \triangleright_{s_2} \tau(t). \end{aligned}$$

Moreover, we assume that $\{t, t'\} \succ_{s_k} \{t_k, t'_k\}$ for $k = 1, 2$. Notice that, this relation is consistent with responsive orders. Preferences of the teachers are:

$$\begin{aligned}
& s_2 P_t s_1 P_t s P_t \emptyset \\
& s_1 P_{t'} s_2 P_{t'} s' P_{t'} \emptyset \\
& s P_{t_1} s_1 P_{t_1} \emptyset \\
& s' P_{t'_1} s_1 P_{t'_1} \emptyset \\
& s P_{t_2} s_2 P_{t_2} \emptyset \\
& s' P_{t'_2} s_2 P_{t'_2} \emptyset
\end{aligned}$$

First note that under any status-quo improving matching, if t is assigned to her first ranked school s_2 , then t' must also be assigned to s_2 . Indeed, let μ be a status-quo improving matching such that $\mu_t = s_2$. Since $\{t, t'\} \succ_{s_2} \{t_2, t'_2\} \succ_{s_2} \{t_2, t\}, \{t'_2, t\}$, status-quo improvement implies that $\mu_{t'} = s_2$. With a similar argument, if $\mu_{t'} = s_1$, then $\mu_t = s_1$. So it implies that there are only three possible SI teacher optimal matchings:

$$\begin{aligned}
\mu^1 &:= \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_1 & s_1 & s & s' & s_2 & s_2 \end{pmatrix} \\
\mu^2 &:= \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_2 & s_2 & s_1 & s_1 & s & s' \end{pmatrix} \\
\mu^3 &:= \begin{pmatrix} t & t' & t_1 & t'_1 & t_2 & t'_2 \\ s_1 & s_2 & s & s_1 & s_2 & s' \end{pmatrix}
\end{aligned}$$

Let φ be a SI teacher optimal mechanism. Assume that $\varphi(P) = \mu^1$. In that case, let $P'_t : s_2 P'_t s P'_t \emptyset$. Under (P'_t, P_{-t}) , the only SI teacher optimal matching is μ^2 so that $\varphi(P'_t, P_{-t}) = \mu^2$ and the manipulation of t is successful. If $\varphi(P) = \mu^2$, then t' can report $P'_{t'} : s_1 P'_{t'} s' P'_{t'} \emptyset$ so that the only SI teacher optimal matching under $(P_{t'}, P_{-t'})$ is μ^1 and $\varphi(P_{t'}, P_{-t'}) = \mu^1$, a successful manipulation for t' . If $\varphi(P) = \mu^3$ then t or t' can manipulate in reporting the same profile as before. We conclude that φ cannot be strategy-proof.

C.2 Immunity to Type Ranking Manipulation

In this section, we investigate whether there exists a status-quo improving, strategy-proof and efficient mechanism which is immune to possible type ranking manipulations. As we explained in our model, we assume schools' preferences, and therefore their type rankings, are commonly known, specifically by policy makers. However, policy makers may choose to report some school s 's type ranking differently to the mechanism in order to improve its match. We say a mechanism ϕ is **immune to type ranking manipulation** if for any P and \triangleright there does not exist a school s and a type ranking \triangleright'_s such that

$$\phi_s(\triangleright'_s, P) \succ_s \phi_s(\triangleright_s, P)$$

where \succsim and \succsim' are preferences induced by type ranking profiles \triangleright and $(\triangleright'_s, \triangleright_{-s})$, respectively. If a mechanism is not immune to type ranking manipulation, then we say it is **vulnerable to type ranking manipulation**.

We first show that there does not exist an SI teacher optimal and strategy-proof mechanism which is immune to type ranking manipulation.

Proposition 7 *Any SI teacher optimal and strategy-proof mechanism is vulnerable to type ranking manipulation.*

Proof: We prove this result by means of an example. On the contrary, suppose there exists a SI teacher optimal and strategy-proof mechanism which is immune to type ranking manipulation. Let ϕ be that mechanism. Let $S = \{s, s', s''\}$, $T = \{t_1, t_2, t_3\}$, the status-quo matching be

$$\omega_s = \{t_1\}, \omega_{s'} = \{t_2\}, \omega_{s''} = \{t_3\},$$

and $q_s = q'_s = q''_s = 1$. Let $\tau(t_2) \triangleright_s \tau(t_3) \triangleright_s \tau(t_1)$, $\tau(t_1) \triangleright_{s'} \tau(t_2)$, and $\tau(t_1) \triangleright_{s''} \tau(t_3)$. Let \succsim be the school preference profile which is induced by \triangleright . The preferences of the teachers are

$$s'' P_{t_1} s' P_{t_1} s$$

$$s P_{t_2} s' P_{t_2} s''$$

$$s P_{t_3} s'' P_{t_3} s'.$$

In this market, there exist two SI teacher optimal matchings:

$$\mu_t = s', \mu_{t'} = s, \mu_{t''} = s''$$

$$\nu_t = s'', \nu_{t'} = s', \nu_{t''} = s.$$

Suppose $\phi(\succsim, P) = \mu$. Let $\tau(t_2) \triangleright'_s \tau(t_1) \triangleright'_s \tau(t_3)$ and \succsim' be the school preference profile induced by $(\triangleright'_s, \triangleright_{-s})$. Then, under market (\succsim', P) ν is the unique SI teacher optimal matching.

Suppose $\phi(\succsim, P) = \nu$. Let $s'' P'_{t_1} s P'_{t_1} s'$. Then, under market $(\succsim, P'_{t_1}, P_{-t_1})$ μ is the unique SI teacher optimal matching. ■

Notice that, we prove Proposition 7 by using a market in which there are at least three types. In many applications, agents are characterized based on two types based on race or gender. In the following proposition, we show that when there are only two types, SI-CC is immune to type ranking manipulation.⁵⁴

⁵⁴If there are at least two new teachers and a school with empty seat, then by ranking one type as unacceptable a school's match can be improved under any SI teacher optimal mechanism.

Proposition 8 When $|\Theta| = 2$ and $|\omega_s| = q_s$ for all $s \in S$, SI-CC is immune to type ranking manipulation.

Proof: On the contrary, suppose there exists a problem (\succsim, P) such that school s can be better off when its type ranking is changed. Let \triangleright induce \succsim and \triangleright'_s be type ranking resulting into improvement for school s . Let $\Theta = \{\theta_1, \theta_2\}$. Without loss of generality, suppose $\theta_1 \triangleright_s \theta_2$. Since teachers in ω_s are with acceptable types, $\theta_1 \triangleright_s \theta_\emptyset$. We consider the following cases.

Case 1: $\theta_2 \triangleright_s \theta_\emptyset$. Then, SI-CC weakly increases the number of θ_1 teachers compared to the one under ω_s . Moreover, all seats will be filled. Under any other type ranking, the number of assigned θ_1 type teachers is at most $|\omega_s^{\theta_1}|$.

Case 2: $\theta_\emptyset \triangleright_s \theta_2$. Then, under SI-CC all assigned teachers to s are with type θ_1 .

In either case, we cannot improve school s by changing its type ranking. ■

Appendix D Additional tables and figures

Figure A.1: AVERAGE TEACHER TYPES

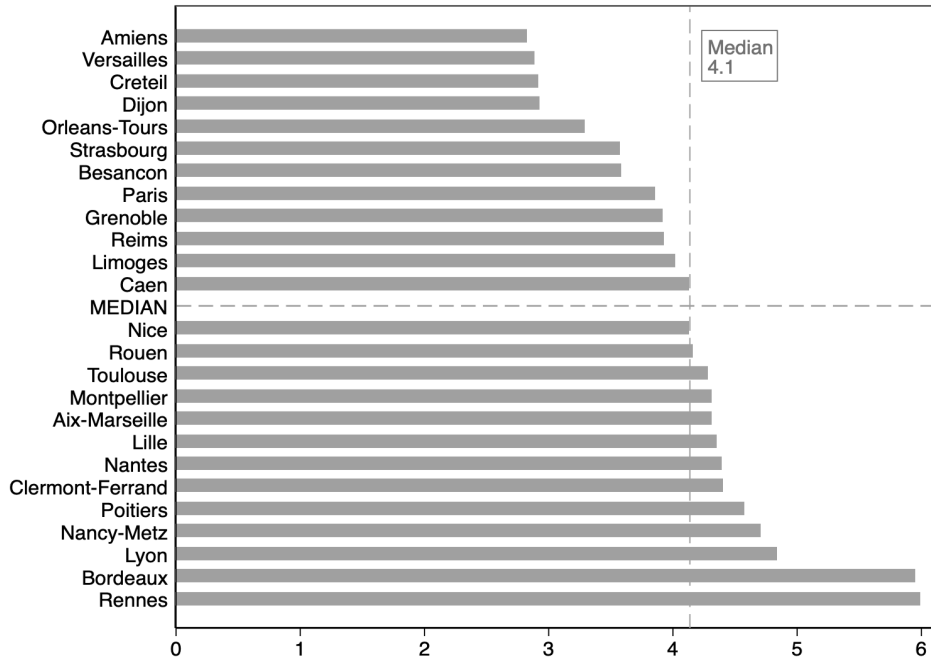
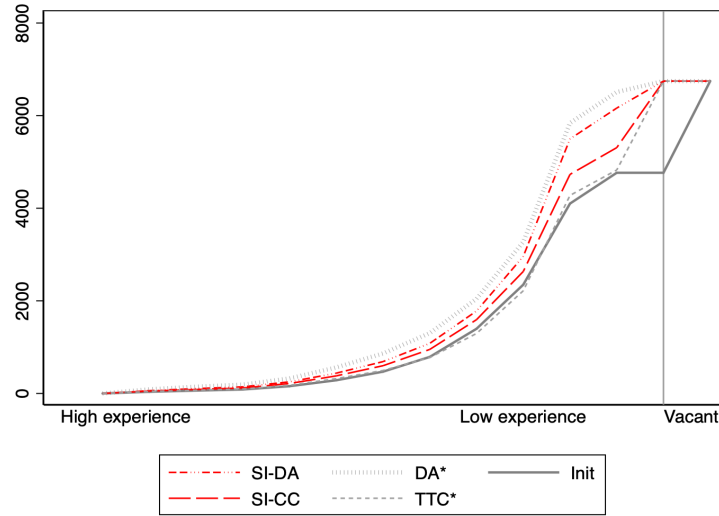
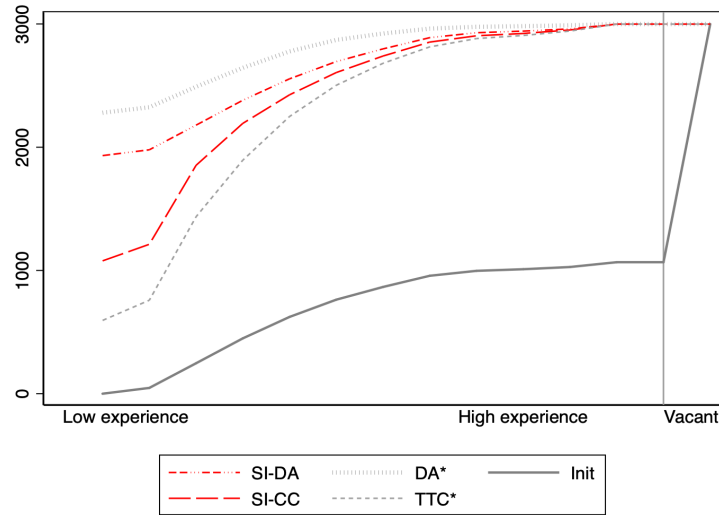


Figure A.2: CUMULATIVE DISTRIBUTION OF TEACHER EXPERIENCE IN THE GROUP OF YOUNG REGIONS



Notes: This Figure shows the cumulative distribution of teachers experience in the youngest regions of France. These are the regions whose average teacher type is strictly lower than the median of teachers types across regions. The vertical axis reports the 12 experience bins of teachers, ordered from most experienced to least experienced. The thirteenth bin corresponds to the vacant positions (the empty type). The mechanisms that respect the FOSD condition are plotted in red. Those that do not are in grey. The thick black line corresponds to the cumulative distribution of teachers' types at the initial assignment.

Figure A.3: CUMULATIVE DISTRIBUTION OF TEACHER EXPERIENCE IN THE GROUP OF OLD REGIONS



Notes: This Figure shows the cumulative distribution of teachers experience in the oldest regions of France. These are the regions whose average teacher type is larger than the median of teachers types across regions. The vertical axis reports the 12 experience bins of teachers, ordered from least experienced to most experienced. The thirteenth bin corresponds to the vacant positions (the empty type). The mechanisms that respect the FOSD condition are plotted in red. Those that do not are in grey. The thick black line corresponds to the cumulative distribution of teachers' types at the initial assignment.

Table A.1: Number of teachers and vacant positions

	All teachers	New teachers	Tenured teachers	Vacant positions
	(1)	(2)	(3)	(4)
All subjects	10460	4627	5833	3912
Sport	2066	568	1498	475
French	1645	786	859	663
English	1374	746	628	640
Mathematics	1563	958	605	824
Spanish	999	316	683	248
History-Geography	1230	657	573	562
Biology	746	286	460	246
Physics-Chemistry	837	310	527	254

Notes: This table reports, for each of the 8 subjects we use for our empirical analysis, the total number of teachers (column 1), the number of teachers who do not have an initial assignment (column 2)—referred to as “new teachers” in the paper—the number of teachers who have an initial assignment (column 3)—referred to as “tenured” in the paper—and the number of vacant positions.

Table A.2: Statistics on regions

Region	Nb tenured teachers asking to enter / leave the region	% of teachers asking for a new assignment coming from each region	Ratio of nb teachers aged more than 50 / less than 30	% students enrolled in “priority education”	% students whose reference parent has no diploma	% students obtaining their baccalaureate
	(1)	(2)	(3)	(4)	(5)	(6)
Rennes	15.55	0.5	8.10	7.9	14.18	91.54
Bordeaux	8.95	0.8	6.56	14.6	19.22	86.25
Toulouse	6.56	1.5	5.29	8.9	17.38	88.57
Paris	3.02	2.8	6.90	25.5	21.54	85.48
Aix-Marseille	2.54	1.9	5.08	30.1	27.20	81.77
Grenoble	1.74	2.3	3.91	16.5	19.80	88.17
Amiens	0.08	6.2	1.89	23.9	27.71	82.41
Créteil	0.03	22.7	1.14	35.5	31.62	83.94
Versailles	0.05	25.7	1.62	24.9	21.88	87.92

This table reports descriptive statistics for the three most attractive regions (Rennes, Bordeaux, and Toulouse), the three least attractive regions (Créteil, Versailles, and Amiens), and three intermediate regions (Paris, Aix-Marseille, and Grenoble). Attractiveness is measured by the ratio of the number of tenured teachers asking to enter a region over the number of teachers asking to leave the region (reported in column 2). All statistics reported in this table come from the following reference: Direction de l’Evaluation de la Prospective et de la Performance (2014).

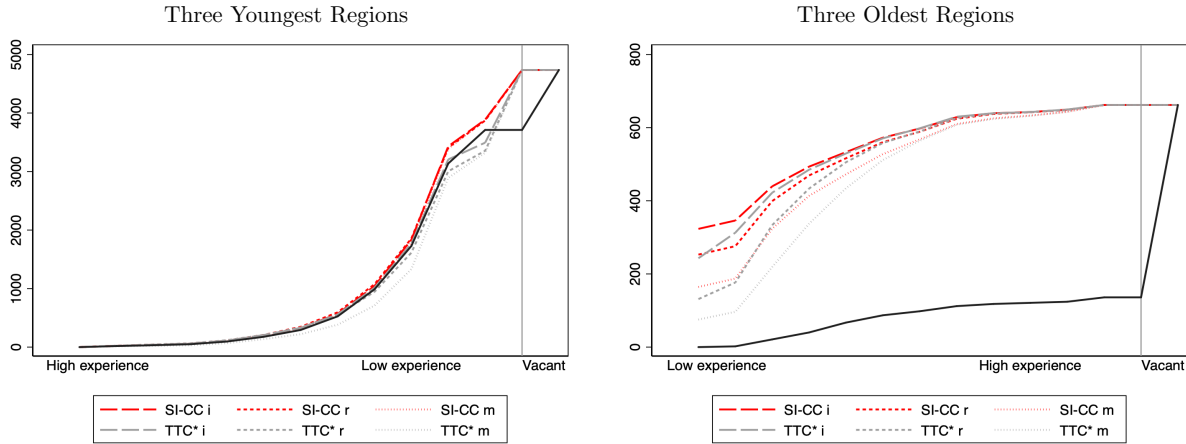
Appendix E Teacher preference estimations

E.1 Variables used for teacher preference estimations

This Appendix describes the variables we use for teacher preference estimations.

We use the following regions’ characteristics:

Figure A.4: CUMULATIVE DISTRIBUTION OF TEACHER EXPERIENCE



Notes: This Figure shows the cumulative distribution of teachers experience. The left panel reports the distribution in the three youngest regions of France (Amiens, Versailles, and Créteil), and the right panel the distribution in the three oldest regions of France (Bordeaux, Rennes, and Lyon). The vertical axis reports the 12 experience bins of teachers, ordered from most experienced to least experienced (left panel) and from least experienced to most experienced (right panel). The thirteenth bin corresponds to the vacant positions (the empty type). The mechanisms that respect the FOSD condition are plotted in red. Those that do not are in grey. The thick black line corresponds to the cumulative distribution of teachers' types at the initial assignment.

- Share of students classified as disadvantaged (labeled as “% disadv stud”).
- Share of students living in an urban area (labeled as “% stud urban”).
- Share of students who attend a school classified as “priority education” (labeled as “% stud in priority educ”). Priority education is a label given to the most disadvantaged schools in France.
- Share of students who attend a private school (labeled as “% stud in private school”).
- Share of teachers who are younger than 30 (labeled as “% teachers younger than 30”).
- Region is in South of France (labeled as “Region in South of France”). The following 5 regions are classified as being in the South of France: Aix-Marseille, Bordeaux, Montpellier, Toulouse, and Nice.

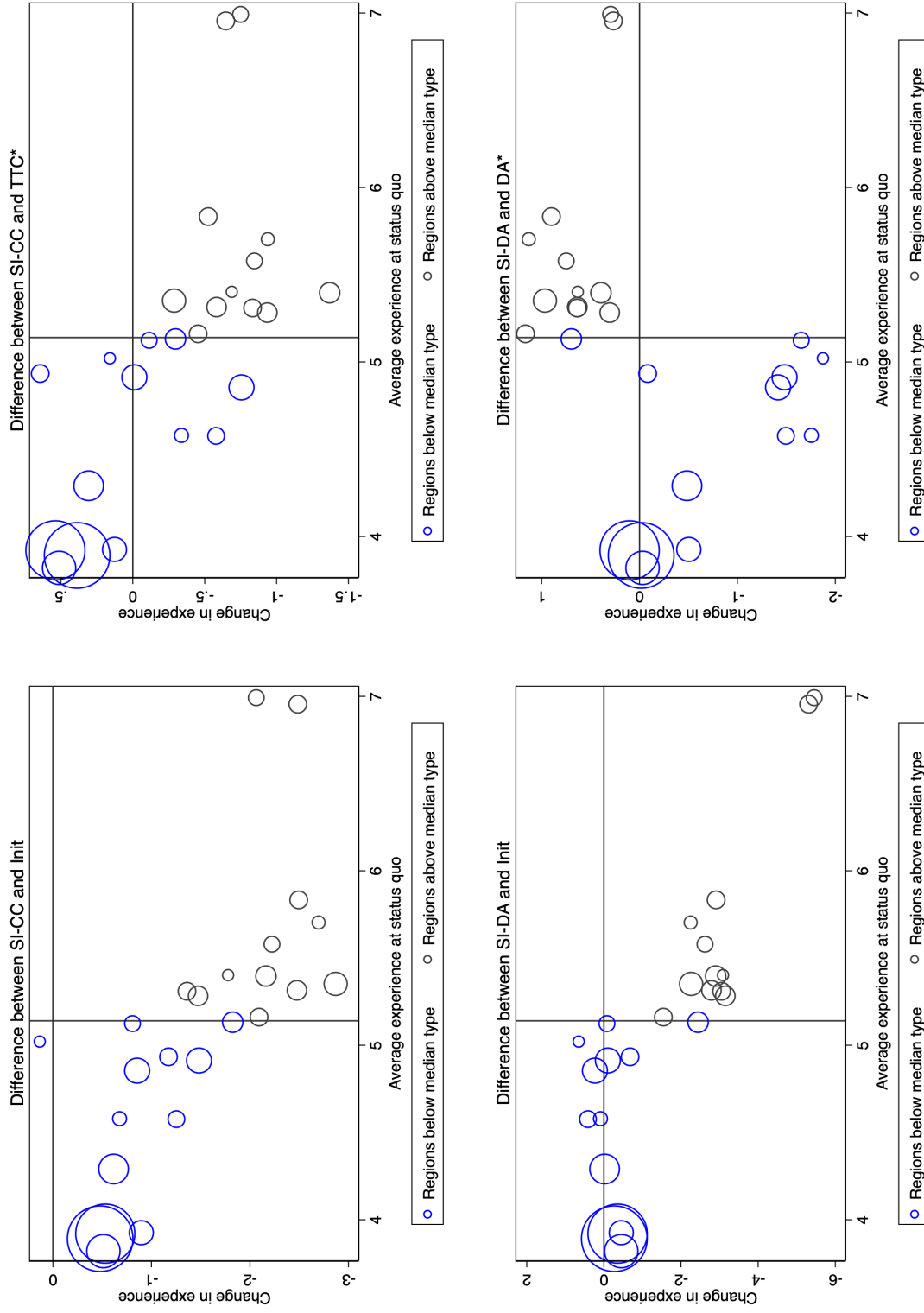
We use the following teachers characteristics:

- Current region of the teacher (labeled as “Current region”). This is the region a teacher is initially assigned to.
- Region where a teacher was born (labeled as “Birth region”).
- Distance between the region ranked and the current region of a teacher (labeled as “Distance current region”).
- Number of years of teaching experience (labeled as “Teach exp”).
- Squared number of years of teaching experience (labeled as “Teach exp sq”).
- Teacher’s current region is Créteil or Versailles, which are the two least attractive regions (labeled as “Teach from CV”). The attractiveness of a region is measured by the ratio of the number of teachers who rank the region divided by the number of teachers who ask to leave

the region.

- Teacher is married (labeled as “Married”).
- Teacher has spent at least 5 years in a school labelled as priority education (labeled as “Teach in disadv sch”).
- Teacher has an advanced teaching qualification (labeled as “Advanced qualif”).

Figure A.5: CHANGE IN (ALL) TEACHER EXPERIENCE (TYPE) ACROSS REGIONS



Notes: This Figure shows the difference in the average experience of teachers (among both tenured and newcomers) between the allocation obtained with SI-CC against the initial allocation (top left figure) and its benchmark TTC* (top right figure). It reports the same difference between the allocation obtained with SI-DA against the initial allocation (bottom left figure) and its benchmark DA* (bottom right figure). Each observation represents a French region. Circle size reflects region size. Regions are ordered (on the x-axis) by average experience of their teachers at status quo (the initial allocation). The vertical line represents the median type. All regions on the left have an average type that is strictly below the median. This is the group of regions we identified as inexperienced regions. All regions on right of the vertical line are regions whose average type is above the median. In regions above the horizontal line, teachers average experience post reassignment is larger than at the initial allocation. The name of the three least experienced regions (Cr eteil, Versailles, and Amiens) and most experienced regions (Rennes, Bordeaux, and Lyon) are reported on the graphs.

Appendix F Description of the French mechanism

The current French mechanism (called DA* in the paper) is a version of the Deferred Acceptance mechanism modified to ensure individual rationality for teachers. Schools preferences are modified such that each teacher t , with an initial assignment s , is ranked in the (modified) ranking of his initial school $s := \mu_0(t)$, above any teacher $t' \notin \mu_0(s)$. Other than this modification, the schools' preference relations remain unchanged. The ministry then runs the (regular) DA mechanism using these modified preferences. See Combe, Tercieux and Terrier (2020) for a more detailed presentation of this mechanism and its properties.⁵⁵

By construction, this mechanism is individually rational for teachers. It does not fulfill FOSD and is not stable under the standard stability notion used in the college admission literature. However, DA* is stable under a weaker stability concept used in Guillen and Kesten (2012), Pereyra, 2013, and Compte and Jehiel, 2008.⁵⁶

⁵⁵Formally, for each school s , a new preference relation \succ'_s is defined such that $t \succ'_s t'$ for each $t \in \mu_0(s)$ and $t' \notin \mu_0(s)$, and for each t, t' not in the school's initial assignment $\mu_0(s)$, we have $t \succ'_s t'$ if and only if $t \succ_s t'$. If $t, t' \in \mu_0(s)$, we assume similarly that these teachers are ranked according to \succ_s .

⁵⁶The intuition behind this notion is that a teacher t would not feel justified envy for a teacher who is initially matched to a school.