

Exclusion of Extreme Jurors and Minority Representation: The Effect of Jury Selection Procedures*

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Abstract

We compare two jury selection procedures meant to safeguard against the inclusion of biased jurors that are perceived as causing minorities to be under-represented. The Strike and Replace procedure presents potential jurors one-by-one to the parties, while the Struck procedure presents all potential jurors before the parties exercise vetoes. Struck more effectively excludes extreme jurors but leads to a worse representation of minorities. The advantage of Struck in terms of excluding extremes is sizable in a wide range of cases. In contrast, Strike and Replace better represents minorities only if the minority and majority are heavily polarized.

JEL Classification: K40, K14, J14, J16

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1 Introduction

In the United States legal system, it is customary to let the parties involved in a jury trial dismiss some of the potential jurors without justification. These dismissals, known as *peremptory challenges*, are meant to enable “each side to exclude those jurors it believes will be most partial toward the other side” thereby “eliminat[ing] extremes of partiality on both sides”.¹ In the last decades, however, peremptory challenges have often been criticized, mainly because they are perceived as causing some groups — in particular minorities — to be under-represented in juries.²

The procedure used to let the parties exercise their challenges varies greatly across jurisdictions and is sometimes left to the discretion of the judge.³ Two classes of procedures are most frequently used. In Struck procedures (henceforth: *STR*), the parties can observe and extensively question *all* the jurors who could potentially serve on their trial *before* exercising their challenges (this questioning process is known as *voir dire*). In contrast, in Strike and Replace procedures (henceforth: *S&R*), smaller groups of jurors are sequentially presented to the parties. The parties observe and question the group they are presented with (sometimes a single juror) but must exercise their challenges on that group *without* knowing the identity of the next potential jurors.

The goal of this paper is to shed light on the debate that emerged in the legal doctrine over the relative effectiveness of *STR* and *S&R* at satisfying the two objectives of excluding extreme jurors and ensuring adequate group representation. [Bermant and Shapard \(1981, pp. 93-94\)](#), for example, argues that, by avoiding uncertainty, *STR* “always gives advocates more information on which to base their challenges, and, therefore, [...] is always to be preferred”. Bermant further notes that “a primary purpose of peremptory challenges is to eliminate extremes of partiality on both sides” and that “the superiority of the struck jury method in accomplishing this purpose is manifest.”

¹*Holland v. Illinois*, 493 U.S. 474, 484 (1990).

²For examples of this line of argument against peremptory challenges, see [Sacks \(1989\)](#), [Broderick \(1992\)](#), [Hochman \(1993\)](#), [Marder \(1994\)](#), and [Smith \(2014\)](#). Despite these attacks, the U.S. has so far resisted abandoning peremptory challenges altogether (unlike other countries; like the U.K., where they were abolished in 1988). Peremptory challenges remain pervasive in all U.S. jurisdictions and have been affirmed by the U.S. Supreme Court as “one of the most important rights secured to the accused” (*Swain v. Alabama* 380 U.S. 202 (1965), see [LaFave et al., 2009](#)).

³For example, in criminal cases in Illinois, “[State Supreme Court] Rule 434(a) expressly grants a trial court the discretion to alter the traditional procedure for impaneling juries so long as the parties have adequate notice of the system to be used and the method does not unduly restrict the use of peremptory challenges” (*People v. McCormick*, 328 Ill.App.3d 378, 766 N.E.2d 671, (2d Dist., 2002)).

Others have argued that, by revealing the identity of all potential jurors before challenges are exercised, *STR* facilitates the exclusion of some groups from juries. In *Batson v. Kentucky*, and *J.E.B. v. Alabama* the Supreme Court found it unconstitutional to challenge potential jurors based on their race or gender.⁴ However, proving that a challenge is based on race or gender is often difficult, and the Supreme Courts’ ruling is therefore notoriously hard to enforce.⁵ Interestingly, in response, judges themselves have turned to the design of the challenge procedure and the use of *SELR* as an instrument to foster adequate group representation. In a memorandum on judges’ practices regarding jury selection, [Shapard and Johnson \(1994\)](#) for example report about judges believing that by “prevent[ing] counsel from knowing who might replace a challenged juror” *SELR* procedures “make it more difficult to pursue a strategy prohibited by *Batson*.”

To inform this debate, we extend in Section 2 the model of jury selection proposed in [Brams and Davis \(1978\)](#) by allowing potential jurors to belong to two different groups. In the model, each potential juror is characterized by a probability to vote in favor of the defendant’s conviction. This probability is drawn from a distribution that depends on the juror’s group-membership. The group distributions are common knowledge but the parties to the trial, a plaintiff and a defendant, only observe their realization for a particular juror upon questioning that juror.

A jury must be formed to decide the outcome of the trial and the parties can influence its composition by challenging (i.e., vetoing) a certain number of potential jurors. Challenges are exercised according to *SELR* or *STR* procedures which, as explained above, differ mainly in the timing of jurors’ questioning (and, as a consequence, in the parties’ ability to observe the conviction probability of potential jurors).

We ask how these two procedures perform in achieving the objectives of excluding extreme jurors and ensuring adequate group representation. In Section 3, we provide some intuition for our main result by introducing an illustrative example where a single juror must be selected, and the parties each have a single challenge available. In this example,

⁴476 U.S. 79 (1986); see also *J. E. B. v. Alabama*, 511 U.S. 127 (1994). The response to these decisions has consisted in allowing the parties to appeal peremptories from their opponent, so that peremptories proven to be based merely on the juror’s race can be nullified. These appeals are known as *Batson appeals*.

⁵See [Raphael and Ungvarsky \(1993\)](#): “In virtually any situation, an intelligent plaintiff can produce a plausible neutral explanation for striking Pat despite the plaintiff’s having acted on racial bias. Consequently, given the current case law, a plaintiff who wishes to offer a pretext for a race-based strike is unlikely to encounter difficulty in crafting a neutral explanation.” See also [Marder \(2012\)](#) or [Daly \(2016\)](#) for why judges rarely rule in favor of *Batson* appeals.

we show that *STR* is more effective than *SEER* at excluding jurors from the tails of the conviction probability distribution, but is less likely to select minority jurors.

The rest of the paper is devoted to characterizing conditions under which these results extend beyond the illustrative example of Section 3. In Section 4 we call a juror *extreme* if its conviction probability falls below (above) a given threshold. We prove that there always exists a low enough threshold such that *STR* is more likely than *SEER* to exclude extreme jurors. Moreover, we show that *STR* always selects fewer extreme jurors than a random selection would, but that there are some (admittedly somewhat unusual) circumstances in which *SEER* would not. Simulations assuming a wide range of conviction probability distributions reveal that, in terms of excluding extreme jurors, the advantage of *STR* over *SEER* can be substantial, even for relatively high thresholds.

Section 5 compares procedures according to their ability to select minorities, identifying conditions under which *SEER* selects more minority jurors than *STR*. Our proof uses a limiting argument showing that the result holds when the minority is vanishingly small and the distributions of conviction probabilities for each group minimally overlap (i.e., groups are polarized). However, simulations suggest that the result remains true when the size of the minority is relatively high and the overlap between distributions is significant.

In Section 6, we explore how changing the number of challenges affects the results of Sections 4 and 5. In any procedure, increasing the number of challenges helps the exclusion of more extreme jurors, but reduces minority representation.

In Section 7 we show how our main theoretical results extend to a different definition of extreme juries (i.e., a jury in which the *highest (lowest)* conviction-probability juror is below (above) a given threshold). We also explore how the procedures compare in selecting members of groups that are of similar sizes (such as males and females, as opposed to minorities which induce groups of unequal sizes).

Related Literature

This paper belongs to a relatively small literature formalizing jury selection procedures. [Brams and Davis \(1978\)](#) model *SEER* as a game and derive its subgame-perfect equilibrium strategies which we use in our theoretical results and simulations. Perhaps closest to our paper is [Flanagan \(2015\)](#) who shows that, compared to randomly selecting jurors, *STR* increases the probability that all jurors come from one particular side of the median of

the distribution of conviction probabilities (because *STR* induces correlation between the conviction probability of the selected jurors). To our knowledge, this literature is silent on the implications of jury selection for group representation and on the trade-off between excluding extreme jurors and ensuring adequate group representation induced by using different procedures. These are the focus and main contributions of this paper.

While the group-composition of a jury has been shown to influence the outcome of a trial (Anwar et al., 2012, 2019, 2021; Flanagan, 2018; Hoekstra and Street, 2021), legal scholars often argue in favor of representative juries regardless of their effect on verdicts. Diamond et al. (2009) for example argue that “unrepresentative juries [...] threaten the public’s faith in the legitimacy of the legal system.” In an experiment on jury-eligible individuals, they show that participants rate the outcome of trials as significantly fairer when the jury is racially heterogeneous than when it is not. This motivates us to consider group-representativity itself as a desirable feature of jury selection procedures.

The empirical literature on jury selection has also identified systematic patterns of group-specific challenges from the parties, with the plaintiffs being almost always more likely to remove minority jurors than defendants (Anwar et al., 2012, 2021; Craft, 2018; Diamond et al., 2009; Flanagan, 2018; Rose, 1999; Turner et al., 1986). This evidence justifies our assumption that parties perceive different groups as having polarized distributions of conviction probabilities.

The lack of random variation in jury selection procedures makes it difficult for the empirical literature to provide credible evidence over the effects of the choice of procedure. Focusing on the number of challenges, Diamond et al. (2009) show that larger juries are more representative of the pool’s demographic.⁶ In Section 6, we show that limiting the number of challenges (while keeping the number of selected jurors fixed) can have a similar effect, though at the expense of a less effective exclusion of extreme jurors.

2 Model

There are two parties to a trial, the **defendant**, D , and the **plaintiff**, P . The outcome of the trial is decided by a jury of j jurors who must be selected from the population. The parties share a common belief about the probability that a juror i will vote to convict the defendant.

⁶The study takes advantage of a feature of civil cases in Florida where juries are made of six jurors unless one of the parties requests a jury of twelve jurors and pays for the costs associated with such a larger jury.

We denote this probability $c_i \in [0, 1]$. Jurors draw this probability independently from the same random variable C , with probability distribution $f(c)$. We denote its cumulative with $F(c)$ and its expected value with μ . Throughout, we assume that C is continuous. To simplify the notation, we also assume that the boundaries of the support of C are 0 and 1.⁷

To address the issue of group representation, we assume that jurors belong to one of two groups, a or b . The parties have common beliefs about the probability that jurors from each group vote to convict the defendant. We index the distributions representing these beliefs and their averages with subscript $g \in \{a, b\}$: $f_g(c)$, $F_g(c)$, and μ_g .⁸ The corresponding random variables are denoted by C_a and C_b . Although throughout conviction probabilities and their distributions across groups should only be viewed as representing the parties common-*beliefs*, we henceforth lighten the terminology and speak directly of conviction probabilities (rather than parties' *beliefs* about conviction probabilities).

We let r denote the proportion of group- a jurors in the population, and when discussing group representation, we assume that C is obtained by drawing from C_a with probability r and from C_b with probability $(1 - r)$ (in particular, $f(c) = rf_a(c) + (1 - r)f_b(c)$).

Following the literature (Brams and Davis, 1978; Flanagan, 2015), we assume that during jury selection the parties do not account for the process of jury deliberations and, perhaps as a way to cope with the complexity of jury selection, view the jurors' conviction probabilities as independent from one another.⁹ Since conviction in most U.S. trials requires a unanimous jury, the parties assume that a jury composed of jurors with conviction probabilities $\{c_i\}_{i=1}^j$ convict the defendant with probability $\prod_{i=1}^j c_i$. The defendant, therefore, aims at minimizing the product $\prod_{i=1}^j c_i$ while the plaintiff wants to maximize it.

To influence the composition of the jury, the defendant and the plaintiff are allowed to challenge (veto) up to d and p of the jurors in a **panel** of $n = j + d + p$ potential jurors randomly and independently drawn from the **population** (sometimes also called the **pool**).¹⁰ To avoid trivial cases, we assume throughout that $d, p \geq 1$. The parties use these challenges in the course of a **veto procedure** M (formally, an extensive game-form). The

⁷This assumption is without loss of generality and all our results hold if C is re-scaled in such a way that $F(c) = 0$ or $[1 - F(1 - c')] = 0$ for some $c, c' \in (0, 1)$.

⁸Empirical evidence shows that that parties use their challenges unevenly across groups (see the Related Literature section of the Introduction).

⁹See Gerardi and Yariv (2007) and Iaryczower et al. (2018) for cases where jury deliberations have an impact on outcomes.

¹⁰In the legal literature, what we call "panel" is sometimes called "*venire*" (though terminology varies and the latter term is sometimes used to speak of what we call the population).

jury resulting from the procedure is called the **effective jury**.

The two veto procedures we study are the **STRuck** procedure (*STR*) and the **Strike And Replace** procedure (*SER*). For comparison, we also consider the **Random** procedure (*RAN*) which simply draws j jurors independently at random from the population. In all procedures, we assume that once a potential juror i is presented to the parties, the parties observe the realized value of c_i for that juror.¹¹ The two procedures however differ in the timing with which jurors are presented to the parties.

Under *STR*, the entire panel of $j+d+p$ potential jurors is presented to the parties *before* they have the opportunity to use any of their challenges. Each party, therefore, observes the value of c_i for every juror in the panel. The defendant and the plaintiff then choose to challenge up to d and p of the jurors in the panel, respectively. In equilibrium, this leads the plaintiff to challenge the p jurors in the panel with lowest conviction probabilities, and the defendant to challenge the d jurors with highest conviction probabilities.¹² Whether these challenges happen simultaneously or sequentially has no impact on the equilibrium of *STR* and our results therefore apply in either case.¹³

Under *SER*, groups of potential jurors are randomly drawn from the population and sequentially presented to the parties. In contrast with *STR* procedures, the parties must exercise their challenges on jurors from a given group *without* knowing the identity of jurors from subsequent groups. There is variation among *SER* used in practice in the size of the groups that are presented in each round.¹⁴ For concreteness and tractability, we focus in this paper on the *SER* procedure in which jurors are presented to the parties *one at a time*. The defendant and the plaintiff start the procedure with d and p challenges left, respectively. After each draw, the plaintiff and the defendant observe the potential juror's

¹¹The assumption that parties have the same assessment of the probability a juror will vote for conviction is motivated by the practice of letting parties extensively question potential jurors in the *voir dire* process. This process typically occurs in the presence of all parties, who therefore have access to the same information about the jurors' demographics, background, and opinions.

¹²Alternative methods used in the field include procedures in which the parties challenge sequentially out of subgroups of jurors from the panel only. As long as the procedure remains of the struck type (i.e., the entire panel — and not only the first subgroup — is questioned before the parties start exercising their challenges), the equilibrium effective jury is often the same as under the *STR* procedure we consider here. Other outcome-irrelevant aspects of the equilibrium might, however, be different such as the number of challenges used by the parties (e.g., if the first group is made of the j “middle” jurors in the panel, they may in some cases be selected as effective jurors without the parties exercising any of their challenges).

¹³Since C is continuous, the probability that two jurors in a panel have the same conviction probability and one of the parties does not use all of its challenges in equilibrium has measure zero and this eventuality can therefore be neglected.

¹⁴As well as in the ability of the parties to challenge, in a later round, potential jurors who were left unchallenged in previous rounds, a practice known as “*backstricking*”.

conviction probability and, if they have at least one challenge left, choose whether or not to challenge the juror. If a juror is not challenged by either party, it becomes a member of the effective jury. Any challenged juror is dismissed and the number of challenges available to the challenging party is decreased by one. The process continues until an effective jury of j members is formed.

The (subgame perfect) equilibrium of SER was characterized by [Brams and Davis \(1978\)](#) and takes the form of threshold strategies. In every subgame, D challenges the presented juror i if c_i is above a certain threshold t_D , P challenges i if c_i is below some threshold t_P , and neither of the parties challenges i if $c_i \in [t_P, t_D]$.¹⁵ We will sometimes refer to these values as *challenge thresholds*. As [Brams and Davis \(1978\)](#) show, in any subgame, $t_P < t_D$ which implies that a challenge to the same juror by both parties never occurs in equilibrium. The equilibrium is therefore unaffected by the order in which the parties decide whether to challenge the presented juror.

In our description of SER , Nature moves in each round by presenting to the parties a new potential juror drawn from the population. To facilitate comparisons between STR and SER , it will sometimes be useful to consider an equivalent description of SER in which Nature first draws a panel of n jurors $\{c_1, \dots, c_n\}$ (which the parties are not aware of) and in each round k presents juror c_k to the parties. For similar purposes, it will sometimes be useful to view RAN as first drawing a panel of n jurors and then (uniformly at random) selecting j jurors among these n to form the effective jury.

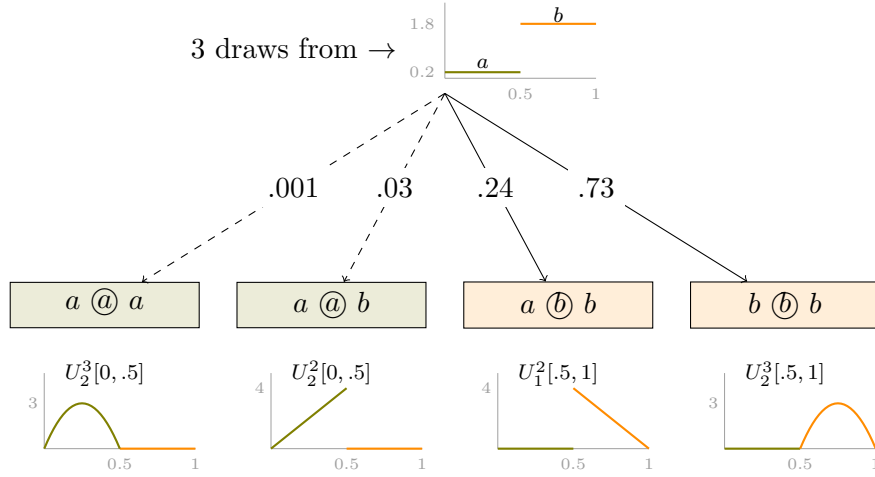
3 Excluding extremes and representation of minorities: An illustrative example

To illustrate the differences between the two procedures, consider the simple case $d = p = j = 1$ together with distributions $C_a \sim U[0, 0.5]$ and $C_b \sim U[0.5, 1]$. Also, suppose that $r = 0.1$, i.e., there is a minority of 10% of group- a jurors in the population.

Let $U_x^n[0, 1]$ denote the x -th order statistic for a $U[0, 1]$ random sample of size n . With this notation, [Figure 1](#) shows the group-membership and distribution of conviction probabil-

¹⁵Each subgame can be characterized by the number of jurors κ that remain to be selected, the number of challenges left to the defendant δ , and the number of challenges left to the plaintiff π . The parties threshold in subgame (κ, δ, π) are a function of the value of subgames $(\kappa - 1, \delta, \pi)$, $(\kappa, \delta - 1, \pi)$, and $(\kappa, \delta, \pi - 1)$ (which are all possible successors to the parties action in (κ, δ, π)) and the distribution of C , see [Brams and Davis \(1978\)](#).

Figure 1: Illustrative example, equilibrium outcomes under *STR*

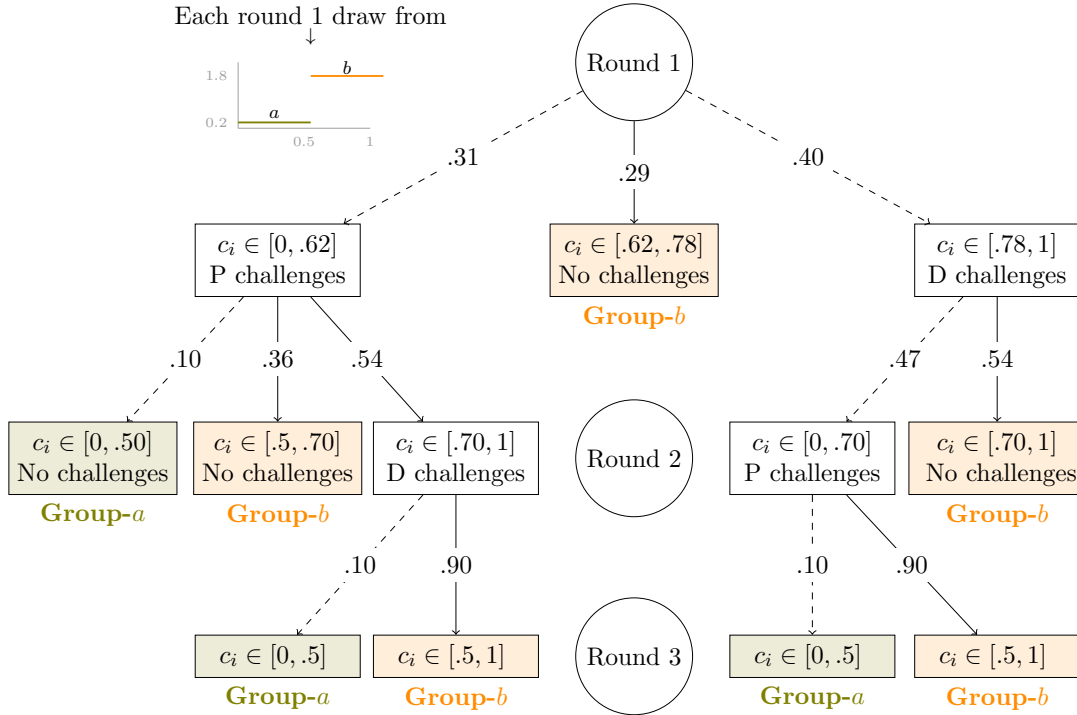


Note: The figure describes the equilibrium of *STR* assuming $j = p = d = 1$, $C_a \sim U[0, 0.5]$, $C_b \sim U[0.5, 1]$, and $r = 0.10$. The initial node illustrates distribution $C = 0.10 * C_a + 0.9 * C_b$. The numbers on each arrow indicate the probability of drawing a panel with the group-composition represented in the pointed boxes (conditional on each panel composition, the circled letter in the box corresponds to the group-membership of the selected juror). Dashed arrows correspond to outcomes that lead to the selection of a group-*a* juror and the graph underneath each box shows the distribution of conviction probabilities for the selected juror.

ity for the juror selected under *STR*, conditional on the composition of the panel. Observe that in this example, if there are group-*a* jurors in the panel, one of them is systematically challenged by the plaintiff. Therefore, for a group-*a* juror (i.e., a minority juror) to be selected under *STR*, there needs to be at least two group-*a* jurors in the panel of $n = 3$ presented to the parties. This occurs with probability 0.03.

In contrast, a group-*a* juror can be selected under *SE \mathcal{R}* even if the panel contains a single group-*a* juror. To understand why, consider the equilibrium of *SE \mathcal{R}* which is illustrated in Figure 2. If a group-*b* prospective juror with a sufficiently low conviction probability ($c_i \in [0.5, 0.62]$) is presented first, then it will be challenged by the plaintiff. This leads to a subgame in which only the defendant has challenges left and a group-*a* juror is more likely to be selected than if a juror was randomly drawn from the population. In particular, any group-*a* juror presented at the beginning of this later subgame is left unchallenged by the defendant and selected to be the effective juror (even if this juror is the only group-*a* juror in the panel because the third juror — who, in this case, is never presented to the parties — happens to be a group-*b* juror). This course of action follows from P 's choice to challenge a group-*b* juror with low conviction probability in the first round, which leaves P without

Figure 2: Illustrative example, equilibrium strategies and outcomes under $S\mathcal{E}R$



Note: The figure describes the equilibrium strategies conditional on the conviction probability of the juror drawn in each round for the case $j = d = p = 1$, $C_a \sim U[0, 0.5]$, $C_b \sim U[0.5, 1]$ and $r = 0.10$. Dashed arrows correspond to paths that may lead to the selection of a group-*a* juror. The numbers on each arrow indicate the probability of the path conditional on reaching the previous node. The second row of text inside boxes indicates an equilibrium action, whereas bold text below boxes indicates the group of the selected juror in the game outcome. In round 3, challenges from both parties are exhausted and the parties do not take any action.

challenges left in the second round. This choice of *P* is optimal from the perspective of the first round of $S\mathcal{E}R$ (before the plaintiff learns that the second juror in the panel is a group-*a* juror), but suboptimal under STR where, having observed the conviction probability of all jurors in the panel, the plaintiff would have challenged the group-*a* juror instead.

Considering only the branch of the $S\mathcal{E}R$ game-tree that starts with a challenge from *P*, the probability of selecting a group-*a* juror is almost $0.05 = 0.31 * (0.54 * 0.1 + 0.10)$. Adding the possibility that a minority juror is selected after *D* challenges in the first round followed by a challenge from *P* in the second round (which happens with probability $0.4 * 0.47 * 0.1 \approx 0.02$), the probability of selecting a minority juror under $S\mathcal{E}R$ is 0.067.¹⁶ This is larger than

¹⁶These are the only cases in which a minority juror can be selected under $S\mathcal{E}R$. In particular, jurors accepted in the first round are always group-*b* jurors ($c_i \in [0.62, 0.78]$). So are jurors accepted in the second round following a challenge from *D* in the first round ($c_i \in [0.70, 1]$).

the probability under *STR*, 0.03, yet smaller than under *RAN*, 0.10.

In this example, the better representation of minority jurors produced by *SEER* comes at the expense of selecting more extreme jurors. Suppose for the sake of illustration that jurors are considered extreme if they come from the top or bottom 5th percentile of C . In our example, the bottom and top 5th percentile corresponds to conviction probabilities below 0.25 and above 0.94, respectively. The selected juror is within the bottom range with probability 0.015 under *STR* versus 0.033 under *SEER*, and in the top range with probability 0.076 under *STR* versus 0.083 under *SEER*.

To understand the source of these differences, consider the bottom 5th percentile $[0, 0.25]$ (a symmetric explanation applies to the *top* 5th percentile). As indicated in Figure 1, when *STR* selects a group- a juror — the type of juror whose conviction probability could possibly be in the bottom 5th percentile — the distribution of that juror’s conviction probability follows the middle or upper order-statistics of a random sample from C_a . These order-statistics are unlikely to result in the selection of a juror with conviction probability in the bottom 5th percentile. In contrast, as Figure 2 illustrates, all paths leading *SEER* to select a group- a juror result in the juror’s conviction probability being drawn from $U[0, 0.5]$ itself, which makes *SEER* more likely to select a juror in the bottom 5th percentile than *STR*.

In the next two sections, we investigate the extent to which the advantages of *SEER* in terms of minority-representation and of *STR* in terms of exclusion of extreme generalizes beyond this illustrative example.

4 Exclusion of extremes

The peremptory challenge procedures implemented in U.S. jurisdictions are often viewed as a way to foster impartiality by preventing extreme potential jurors from serving on the effective jury.¹⁷ In the context of our model, we interpret this goal as that of limiting the presence in the jury of jurors from the tails of the distributions of conviction probabilities.

We define a juror i as *extreme* if its conviction probability c_i lies below or above given thresholds (see Section 7 for results under an alternative definition). For brevity, we will focus on jurors who qualify as extreme because their conviction probability lies *below* some

¹⁷See Footnote 1 and its associated quote. For legal arguments in favor of peremptory challenges based on the Sixth Amendment, see, among others, Beck (1998), Biedenbender (1991), Bonebrake (1988), Horwitz (1992), and Keene (2009).

threshold $\underline{c} > 0$. All our results about extreme jurors apply symmetrically to jurors whose conviction probability lies *above* a given threshold $\bar{c} < 1$.

In our example from the previous section, jurors in the bottom 5th percentile of C are selected less often under STR than SER . This is not true in general. Fixing a particular threshold $\underline{c} > 0$ — or percentile of C — to characterize jurors as extreme, there always exists distributions of C and values of d , p , and j such that SER selects fewer extreme jurors than STR . However, our first result shows that regardless of the distribution and of the parameter values, there always exists a sufficiently small threshold such that the probability of selecting extreme jurors (i.e., below that threshold) is greater under SER than under STR .

Let $\mathbb{T}_M(x; c)$ denote the probability that there are at least x jurors with conviction probability *smaller or equal* to c in the jury selected by procedure M .

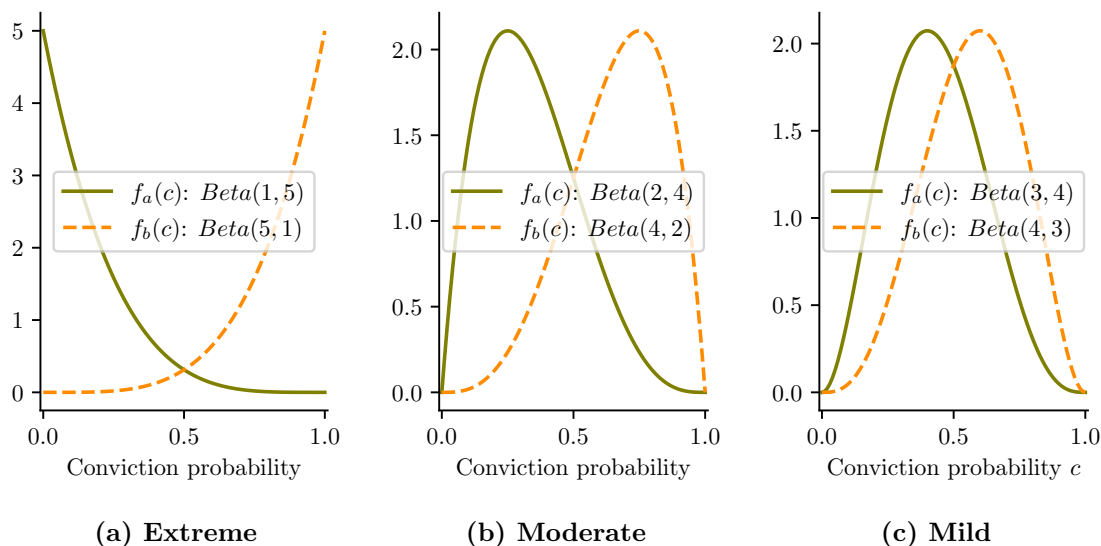
Proposition 1. *For any $x \in \{1, \dots, j\}$, there exists $\underline{c} > 0$ such that $\mathbb{T}_{STR}(x; c) < \mathbb{T}_{SER}(x; c)$ for all $c \in (0, \underline{c})$.*

All proofs are in the appendix. A symmetric statement, which we omit, applies for extreme jurors at the right-end of the distribution. Note that Proposition 1 can be rephrased in terms of stochastic dominance. Let N_M^c denote the expected number of jurors of type $c_i \leq c$ in the jury selected by procedure M . Then, Proposition 1 says that there exists $\underline{c} > 0$ and such that N_{SER}^c has first-order stochastic dominance over N_{STR}^c for all $c \in (0, \underline{c})$. A direct corollary of Proposition 1 is therefore that the *expected number* of extreme jurors is larger under SER than under STR .

For some intuition about Proposition 1, consider the case $x = 1$. As illustrated in Section 3, the panel must be composed of more than one extreme juror for STR to select at least one such juror (since, if there is a single extreme juror in the panel, that juror is systematically challenged by the plaintiff). In contrast, even in panels with a single extreme juror, the extreme juror can be part of the effective jury resulting from SER . This happens, for example, if the extreme juror is presented to the parties after they both exhausted all their challenges. The single extreme juror can also be accepted by both parties if its conviction probability is sufficiently close to \underline{c} and it is presented after the plaintiff used most of its challenges on non-extreme potential jurors.¹⁸ The proof then follows from the

¹⁸Subgames in which the defendant has more challenges left than the plaintiff can lead the plaintiff to be conservative and accept jurors who are “barely extreme” ($c_i \approx \underline{c}$) in order to save its few challenges left for “very extreme” jurors ($c_i \approx 0$).

Figure 3: Distributions of conviction probabilities by group under extreme, moderate, and mild group-polarization

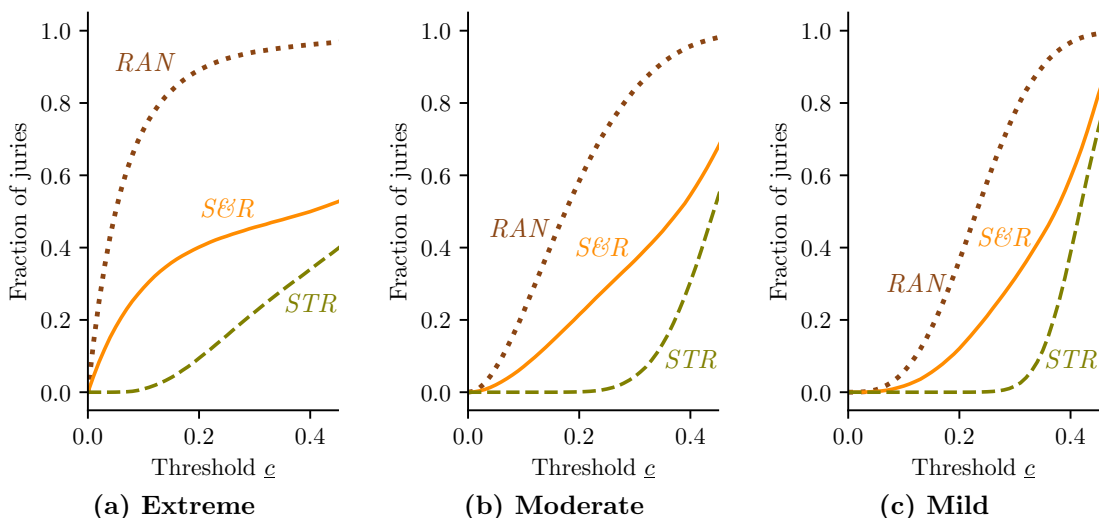


fact that, as \underline{c} tends to zero, the probability that the panel contains more than one extreme juror goes to zero faster than the probability the panel contains a single extreme juror.¹⁹

Proposition 1 is silent about the value of the threshold \underline{c} below which STR selects fewer jurors than SER , as well as the size of $\mathbb{T}_{SER}(x; c) - \mathbb{T}_{STR}(x; c)$ for $c < \underline{c}$. These values depend on the model's parameters. To illustrate, we simulate $\mathbb{T}_{STR}(1; c)$ and $\mathbb{T}_{SER}(1; c)$ using $j = 12$, $d = 6$, and $p = 6$, a typical combination of jury size and number of peremptory challenges in U.S. jurisdictions. For the distribution of conviction probabilities in the population, we use symmetric mixtures of beta distributions that represent a population made of two groups with polarized views. Although the results in this section do not depend on whether jurors come from polarized groups, using these distributions facilitates comparisons with Section 5 where we study group-representation. We provide simulation results for three mixtures of the distributions illustrated in Figure 3, which are meant to represent extreme (Panel (a)), moderate (Panel (b)), and mild levels of polarization (Panel (c)). Additional simulations

¹⁹Proposition 1 crucially depends on averaging across all possible panels and does *not* state that STR rejects more extreme jurors than SER for *any* particular realization of the panel. The latter would obviously imply Proposition 1 but turns out to be false in general. For a counterexample, let $j = d = p = 1$. Consider a panel of three jurors with $c_2 < c_3 < \underline{c}$ and $c_1 > \underline{c}$ and where the index of the jurors indicate the order in which they are presented under SER . For this panel, STR always leads to the selection of extreme juror 3. In contrast, provided c_2 falls between the challenge thresholds of the defendant and the plaintiff in the first round (which happens with positive probability), SER selects non-extreme juror 2.

Figure 4: Fraction of juries with at least one extreme juror



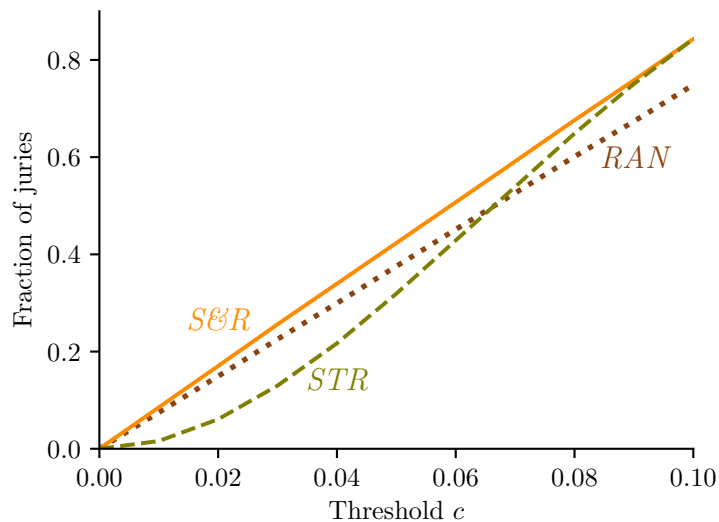
Note: For each set of parameters, results on the vertical axis are averages across 50,000 simulated jury selections, fixing $j = 12$, $d = p = 6$, and $C \sim 0.25 * C_a + 0.75 * C_b$ throughout (distributions C_a and C_b illustrated in Figure 3). Each line illustrates the fraction of juries with at least one extreme juror, where a juror is considered extreme if her conviction probability falls below the threshold c corresponding to the value on the horizontal axis.

results using $U[0, 1]$ are reported in the external appendix ??.

Using these parameters, *STR* is found to exclude more extreme jurors than *S&E* even when the threshold for defining jurors as extreme is relatively high. As illustrated in Figure 4, the difference between the propensity of *STR* and *S&E* to select extreme jurors is sizable. For example, in all three sets of simulations, less than 1% of juries selected by *STR* feature include at least one juror with conviction probability below the 10th percentile of the distribution (the 10th percentile corresponds to 0.10 under the extreme polarization distribution, 0.25 under moderate polarization, and 0.28 under mild polarization). Under *S&E*, the proportion of juries with at least one juror below the 10th percentile rises to 29% with extreme polarization, 28% with moderate polarization, and remains quite high at 27% even under mild polarization. For comparison, a random selection would have resulted in over 70% of the juries featuring at least one such juror in all scenarios.

In these simulations, both procedures select fewer extreme jurors than a random draw from the population. Somewhat surprisingly, this is not true in general. There exist distributions and values of the parameters d , p and j for which *S&E* selects *more* extreme

Figure 5: Fraction of juries with at least one extreme juror (case in which $S\mathcal{E}R$ is more likely to pick extreme jurors than RAN)



Note: For each set of parameters, results on the vertical axis are averages across 50,000 simulated jury selections, fixing $j = d = p = 1$, and $C \sim 0.75 * U[0, 0.1] + 0.25 * U[0.9, 1]$ throughout. Each line illustrates the fraction of juries with at least one extreme juror, where a juror is considered extreme if her conviction probability falls below the threshold \underline{c} corresponding to the value on the horizontal axis.

jurors than RAN , no matter how small the threshold below which a juror is considered as extreme. In contrast, as we show in the next proposition, STR always selects fewer extreme jurors than RAN .

Proposition 2. *For any $x \in \{0, \dots, j - 1\}$, there exists $\underline{c} > 0$ such that $\mathbb{T}_{STR}(x; c) < \mathbb{T}_{RAN}(x; c)$ for all $c \in (0, \underline{c})$.*²⁰

Figure 5 illustrates Proposition 2 and the fact that a similar statement does not hold for $S\mathcal{E}R$. For the simulations in the figure, we let $j = d = p = 1$ and adopt an extremely polarized distribution of conviction probabilities with $C \sim 0.75 * U[0, 0.1] + 0.25 * U[0.9, 1]$. In this case (as in others), STR excludes extreme jurors more often than RAN because, for any realization of the panel, the juror with the lowest conviction probability is never selected under STR (whereas the same juror is selected with positive probability under RAN). Under $S\mathcal{E}R$, however, if the distribution is sufficiently right-skewed, the plaintiff is more likely than the defendant to challenge in the first round. A challenge by the plaintiff

²⁰Proposition 2 generalizes Theorem 2 in Flanagan (2015) which shows that there always exists $\underline{c} > 0$ such that $\mathbb{T}_{STR}(n; c) < \mathbb{T}_{RAN}(n; c)$ for all $c \in (0, \underline{c})$.

in the first round leads to a subgame in which only the defendant has challenges left and the selection of an extreme juror is more likely than under a random draw. When they are sufficiently large (i) the added probability of selecting an extreme juror when the defendant has more challenges left than the plaintiff, coupled with (ii) the probability of a challenge by the plaintiff in the first round can, as in the simulation depicted in Figure 5, lead to SER selecting more extreme jurors than RAN .

We could not fully characterize the situations in which SER selects more extreme jurors than RAN , and we never observed such a situation in simulations where C is a symmetric mixture of beta or uniform distributions. The example in Figure 5 (as well as other examples we found) requires extreme skewness in the distribution, which may be viewed as unlikely. In this sense, situations in which SER selects *more* extreme jurors than RAN might represent worst-case scenarios for SER 's ineffectiveness at excluding extreme jurors.

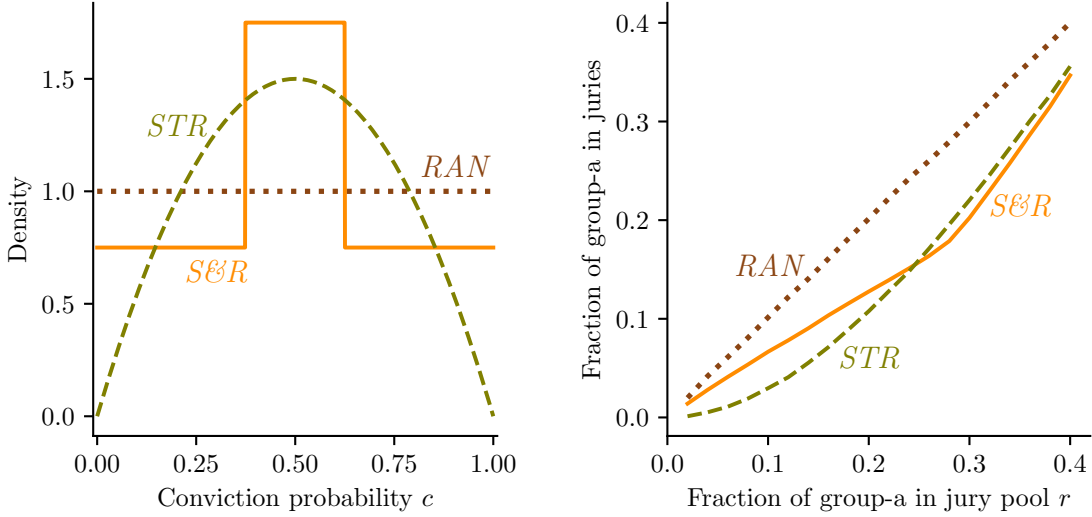
5 Representation of minorities

In this section, we study the extent to which STR 's tendency to exclude more extreme jurors than SER impacts the representation of minorities under the two procedures. Without loss of generality, we let group- a be the minority group. Since the parties do not care intrinsically about group-membership, any asymmetry in the use of their challenges arises from heterogeneity in preferences for conviction between groups. In our simulations, we assume that group- a is biased in favor of acquittal in the sense that C_b first-order stochastically dominates C_a .²¹

As suggested by Proposition 1, which procedure better represents minorities strongly depends on the polarization between the two groups, and the concentration of minority jurors at the tails of the distribution of conviction probabilities. To illustrate, suppose that $d = p = j = 1$ and $C \sim U[0, 1]$. For this case, the distributions of conviction probabilities for the juror selected under RAN , STR , and SER are displayed in Figure 6(a). Consistent with Proposition 1, below some threshold $\underline{c} \approx 0.25$, the probability of selecting a juror i with $c_i < \underline{c}$ is lower under STR than under SER . If the two groups are polarized and the distribution of C_a is sufficiently concentrated below \underline{c} , it follows that STR selects a minority juror less often than SER . But the same is not true if the distributions lack polarization

²¹We also simulated the scenario in which the minority is biased towards conviction, the results, which we report in the Appendix, are symmetrically very close.

Figure 6: Jury selection and minority representation in size-1 juries



(a) Distribution of c for selected juror

(b) Minority representation in juries

Note: For each set of parameter, results on the vertical axes are averages across 50,000 simulated jury selections, fixing $j = 1$, $d = p = 1$, and $C \sim r * U[0, r] + (1 - r) * U[r, 1]$ throughout. The distribution in panel (a) is independent of r ; the lines in panel (b) interpolate results from 20 values of r .

or the minority is too large. For example, let $C_a \sim U[0, r]$ and $C_b \sim U[r, 1]$ so that $C \sim U[0, 1] = rU[0, r] + (1 - r)U[r, 1]$. Since the parties only care about a juror’s conviction probability and not about its group-membership *per se*, the value of r does not affect the distributions of conviction probabilities for the juror selected under *RAN*, *STR*, or *S&R*. However, as illustrated in Figure 6(b), low values of r — which concentrate minorities at the bottom of the distribution — make *S&R* select more minorities than *STR*, whereas higher values of r — which spread the minority over a larger range of conviction-types — make *STR* select more minorities than *S&R*.

From this example, we see that non-overlapping group-distributions are not sufficient to guarantee that *S&R* selects more minority jurors than *STR*. Neither is making the minority arbitrarily small. For example, regardless of the size of the minority r , concentrating the support of the minority distribution inside the interval $[0.2, 0.3]$ would result in *STR* selecting more minorities, as can be seen from Panel 6(a). However, combining a small minority with group-distributions that minimally overlap concentrates the distribution of group- a at the tails which, as implied by Proposition 1, makes *S&R* select more minorities than *STR*.

Formally, consider a sequence of triples $\{(C_a^i, C_b^i, r^i)\}_{i=1}^\infty$. If,

- (i) $r^i \in (0, 1]$ for all $i \in \mathbb{N}$ with $\lim_{i \rightarrow \infty} r^i = 0$, and
- (ii) C_a^i and C_b^i converge in distribution to C_a^* and C_b^* , with either $\mathbb{P}(C_a^* < C_b^*) = 0$ or $\mathbb{P}(C_a^* > C_b^*) = 0$,

then we say that **there is a vanishing minority and group-distributions that do not overlap in the limit**. For any such sequence, let $\mathbb{A}_M^i(x)$ denote the probability that there are at least x minority jurors in the jury selected by procedure M when group-distributions are C_a^i and C_b^i and the proportion of minority jurors in the population is r^i .

Proposition 3. *Suppose that, under $\{(C_a^i, C_b^i, r^i)\}_{i=1}^\infty$, there is a vanishing minority and group distributions that do not overlap in the limit. Then for all $x \in \{1, \dots, j\}$, there exists k sufficiently large such that $\mathbb{A}_{S\mathcal{E}R}^i(x) > \mathbb{A}_{STR}^i(x)$ for all $i > k$.²²*

Given the result in Proposition 3, it is natural to wonder how small the minority and the overlap between the group-distributions must be for $S\mathcal{E}R$ to select more minority jurors than STR . When the latter is true, one may also wonder about the size of $\mathbb{A}_{S\mathcal{E}R}(x; r) - \mathbb{A}_{STR}(x; r)$ is. Again, the answer depends on the model’s parameters. To inform these questions, we ran a set of simulations with $d = p = 6$ and $j = 12$ using the distributions displayed in Figure 3, where the green lines in each panel represent f_a and the yellow lines f_b .

The results of our simulations, displayed in Table 1, suggest that $S\mathcal{E}R$ might select more minority jurors than STR even when the size of the minority is relatively high (as high as 25%) and the overlap between the group-distributions significant. However, without stark polarization across groups,²³ differences between the procedures’ propensities to select minority jurors appear to be small. For example, under the distributions we labeled as “extreme group heterogeneity” and with minorities representing 10% of the population, only 2.3% of juries selected by $S\mathcal{E}R$ include at least one minority juror whereas this number rises to 17.1% under $S\mathcal{E}R$ (random selection would generate over 70% of such juries). However,

²²Note that, despite the argument presented in the motivating example illustrated in Figure 6, Proposition 3 does not follow directly from Proposition 1. The reason is that, unlike in the motivating example, most of the sequences $\{(C_a^i, C_b^i, r^i)\}_{i=1}^\infty$ covered by Proposition 3 are such that $C^i = r^i C_a^i + (1 - r^i) C_b^i$ varies across the sequence (i.e., $C^h \neq C^k$ for most $h, k \in \mathbb{N}$).

²³Recall that C_a and C_b represent the parties’ beliefs that randomly drawn group- a or group- b jurors eventually vote to convict the defendant. Polarized C_a and C_b , therefore, corresponds to groups that are perceived by the parties to have different probabilities of voting for conviction (whether or not this materializes when jurors actually vote on conviction at the end of the trial).

Table 1: Representation of Group-a when Group-a is a minority of the pool

Polarization Procedure	Extreme		Moderate		Mild		(All)
	<i>SER</i>	<i>STR</i>	<i>SER</i>	<i>STR</i>	<i>SER</i>	<i>STR</i>	<i>RAN</i>
Average fraction of minorities	0.10	0.08	0.18	0.16	0.23	0.23	0.25
Standard deviation	0.11	0.11	0.12	0.12	0.12	0.12	0.12
Fraction of juries with at least 1	0.57	0.45	0.88	0.84	0.96	0.95	0.97

(a) Group-a represents 25% of the jury pool

Polarization Procedure	Extreme		Moderate		Mild		(All)
	<i>SER</i>	<i>STR</i>	<i>SER</i>	<i>STR</i>	<i>SER</i>	<i>STR</i>	<i>RAN</i>
Average fraction of minorities	0.02	0.00	0.05	0.04	0.09	0.08	0.10
Standard deviation	0.04	0.01	0.07	0.06	0.08	0.08	0.09
Fraction of juries with at least 1	0.17	0.02	0.47	0.38	0.67	0.64	0.72

(b) Group-a represents 10% of the jury pool

Note: The rows report the average number and standard deviation of group-*a* jury members, and the percent of juries with at least one group-*a* jurors, out of 50,000 simulations of jury selection with parameters $j = 12$ and $d = p = 6$. Conviction probabilities are drawn from $Beta(5, 1)$, $Beta(1, 5)$, respectively for Group-*a*, Group-*b* jurors (Extreme), from $Beta(4, 2)$, $Beta(2, 4)$ (Moderate), and from $Beta(4, 3)$, $Beta(4, 3)$ (Mild); see Figure 3 for the shape of these distributions.

under the distributions we labeled as “mild group heterogeneity”, the same numbers become 66.5% under *SER* and 64.5% under *STR* (random selection would generate over 71.9% of juries with at least one minority juror in this second case).

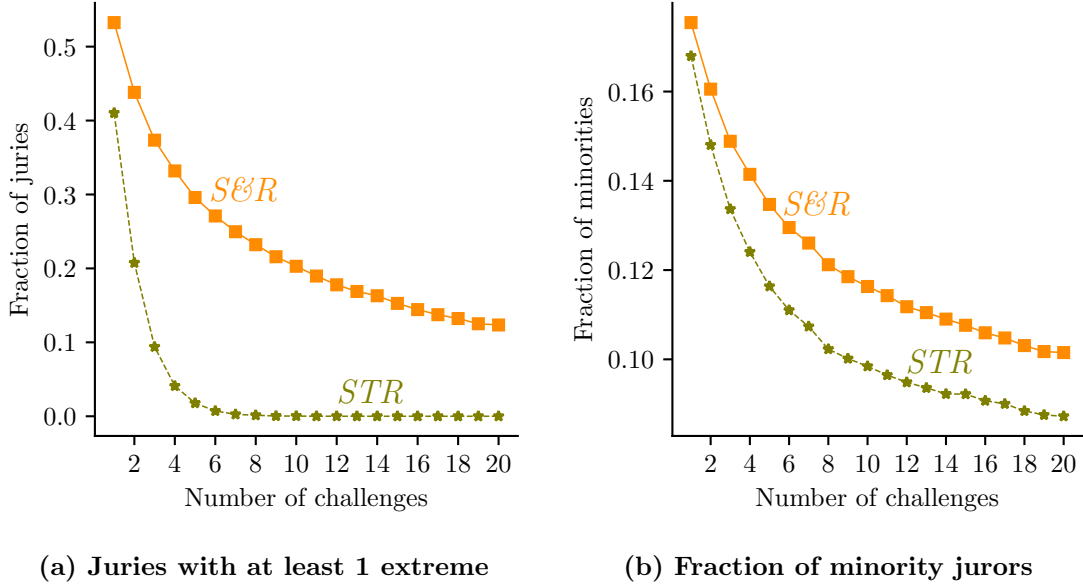
6 Changing the number of challenges

The number of challenges that the parties can use are typically specified by state rules of criminal procedure. In the last decades, several states have reduced the number of challenges the parties can use.²⁴ In some instances, these reforms also clarify or alter the jury selection procedures used in the state.²⁵ In the context of such broader reforms, it is natural to ask how the ability to change *both* the number of challenges the parties are entitled to *and* the

²⁴Examples include California’s Senate Bill 843, passed in 2016, which reduces the number of challenges a criminal defendant is entitled to from 10 to 6 (for charges carrying a maximal punishable of one year in prison, or less).

²⁵Examples include the 2003 reform of jury selection in Tennessee where some aspects of the jury selection procedure were codified to apply uniformly across the state, while the number of peremptory challenges was also slightly reduced (see [Cohen and Cohen, 2003](#)).

Figure 7: The effect of varying the number of challenges



Note: Fraction of juries with at least one juror below the 10th percentile (left panel) and fraction of minority jurors (right panel) under *STR* (green starred markers) and *S&R* (orange square markers). For each set of parameters, results on the vertical axes are averages across 50,000 simulated jury selections, fixing $j = 12$ and $C \sim 0.2 * C_a + 0.8 * C_b$ throughout (distributions $C_a \sim \text{Beta}(2, 4)$ and $C_b \sim \text{Beta}(4, 2)$, see Figure 3(b)). The values of $d = p$ are on the horizontal axes.

procedure through which the parties exert their challenges affect the trade-off between the exclusion of extreme jurors and the representation of minorities.

Throughout this section, we fix an arbitrary value of j and consider varying $d = p$. For any procedure M , let $M-y$ denote the version of M when $d = p = y$. The notation for the two previous sections then carries over, with $\mathbb{T}_{M-y}(x; c)$ denoting the probability that at least x jurors with conviction probability below c are selected under $M-y$, and $\mathbb{A}_{M-y}(x)$ the probability that at least x minority jurors are selected under $M-y$.²⁶

For illustration, we first consider the case $C \sim 0.2 * C_a + 0.8 * C_b$, with $C_a \sim \text{Beta}(2, 4)$ and $C_b \sim \text{Beta}(4, 2)$ (C_a and C_b are illustrated in the Figure 3(b)), and consider a juror as extreme if its conviction probability falls in the bottom 10th percentile of C (0.27). Unsurprisingly, the fraction of juries with at least one *extreme* jurors *decreases* as the number of challenges awarded to the parties increases, regardless of the procedure that is

²⁶Again, in the case of extreme jurors, we focus on jurors who qualify as extreme because their conviction probability falls *below* a certain threshold \underline{c} , though all of our results hold symmetrically for jurors who qualify as extreme because their conviction probability lies *above* a certain threshold \bar{c} ,

used (Figure 7(a)). Conversely, the fraction of *minority* jurors *decreases* with the number of challenges under both procedures (Figure 7(b)). For both *STR* and *SEER*, more challenges lead to fewer extreme jurors being selected at the expense of a lower minority representation.

As Figure 7(a) illustrates, however, increasing the number of challenges decreases the selection of extreme jurors much faster under *STR* than under *SEER*. As a consequence, for all values of $y \in \{2, \dots, 18\}$, there exists $w < y$ such that *STR-w* performs better than *SEER-y* in terms of *both* objectives.²⁷

The latter is not true in general. Even when there exists w such that *STR-w* better represents minorities than *SEER-y*, *STR-w* might still exclude fewer extreme jurors than *SEER-y* if jurors are considered extreme when their conviction probability falls below an arbitrary $c > 0$. However, an extension of Proposition 1 shows that when such a w exists, there also exists $\underline{c} > 0$ such that if jurors are considered extreme when their conviction probability falls below \underline{c} , *STR-w* performs better than *SEER-y* in terms of both objectives.

Proposition 4. *Consider any $x \in \{1, \dots, j\}$ and any $y \geq 1$. Suppose that there exists $w \geq 1$ such that $\mathbb{A}_{STR-w}(x) > \mathbb{A}_{SEER-y}(x)$. Then for some $\underline{c} > 0$, we also have $\mathbb{T}_{STR-w}(x; c) < \mathbb{T}_{SEER-y}(x; c)$ for all $c \in (0, \underline{c})$.*

7 Extensions

7.1 Excluding unbalanced juries

The primary purpose of jury selection is to prevent extreme jurors from serving (see Footnote 1). In our model, it seems natural to interpret this goal as that of limiting the selection of jurors coming from the tail of the distribution, as we have done so far. Another approach is to consider the extremism of juries *as a whole*. For example, extreme juries could be juries in which the juror with the highest or lowest conviction probability is extreme. Using variants of the arguments in the proofs of Propositions 1 and 2, one can show that, in that sense too, *STR* is more effective than both *SEER* and *RAN* at excluding extreme juries.²⁸

²⁷Specifically, in this example, for any $y \in \{2, \dots, 18\}$, there exists $w \in \{1, \dots, y-1\}$ such that $\mathbb{A}_{STR-w}(1) > \mathbb{A}_{SEER-y}(1)$ and $\mathbb{T}_{STR-w}(1; 0.27) < \mathbb{T}_{SEER-y}(1; 0.27)$.

²⁸Specifically, for any $x \in \{0, \dots, j-1\}$, there exists $\underline{c} > 0$ and $\bar{c} < 1$, such that (a) for every $c \in (0, \underline{c})$, the probability that the lowest conviction-probability in the jury is smaller than c is larger under *SEER* and *RAN* than under *STR*, and (b) for every $c \in (\bar{c}, 1)$, the probability that the highest conviction-probability in the jury is larger than c is larger under *SEER* and *RAN* than under *STR*.

Another measure of juries’ extremism, proposed by Flanagan (2015), is whether a jury is excessively “unbalanced” in the sense of featuring a disproportionate proportion of jurors coming from one side of the median of C . Interestingly, Flanagan shows that STR introduces correlation between the selected jurors, which leads the procedure to select more unbalanced juries than RAN . Even though panels are the result of independent draws from the population, jurors selected under STR have conviction probabilities between that of the lowest and highest challenged juror. For example, the selection of two jurors with conviction probabilities 0.25 and 0.75 indicates that challenges were used on jurors with conviction probabilities outside the $[0.25, 0.75]$ range. The latter makes it more likely that STR selected additional jurors between $[0.25, 0.75]$, introducing a correlation between selected jurors.

This intuition is formalized in Corollary 2 of Flanagan (2015) which shows that, even when the parties have the same number of challenges ($d = p$), the probability that *all* selected jurors come from one side of the median is *larger* under STR than under RAN . Our next proposition generalizes this result. Using a new proof technique, we show that for *any* x larger than half the jury-size, the probability of selecting at least x jurors from one side of the median is larger under STR than under RAN . As in Section 4, we focus on the probability that the selected jurors are *below* the median (our results apply symmetrically to selection of jurors above the median). Let $med[C]$ denote the median of C .

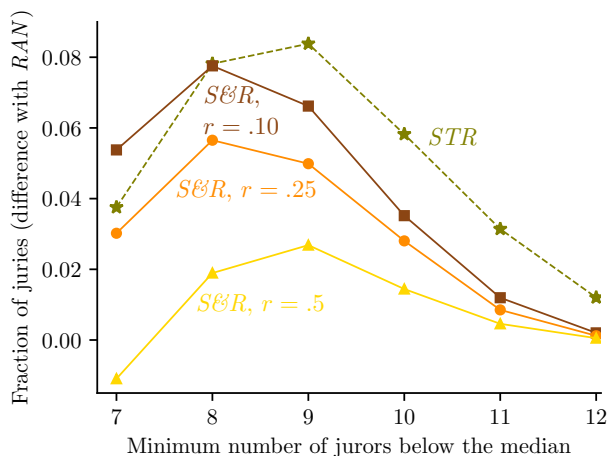
Proposition 5. *If $d = p$, then for any $x \in \{n/2 + 1, \dots, n\}$ if n is even, and any $x \in \{n/2 + 1.5, \dots, n\}$ if n is odd, we have $\mathbb{T}_{STR}(x; med[C]) > \mathbb{T}_{RAN}(x; med[C])$.*

Figure 8 illustrates Proposition 5 and the fact that a similar statement does not hold for SER . For $M \in \{STR, RAN\}$, the value of $\mathbb{T}_M(x; med[C])$ can be computed analytically and does not depend on the distribution of C .²⁹ For $M = SER$, the value of $\mathbb{T}_M(x; med[C])$ depends on the distribution in a complex fashion and it is not possible to generally compare SER with the two other procedures in terms of $\mathbb{T}_M(x; med[C])$. As the figure illustrates, the fraction of simulated juries with at least x jurors below $med[C]$ can, in some cases (in the figure, $x = 7$ and, barely, $x = 8$ jurors), be larger under SER than under both RAN and STR . In other cases, however, the same figure is lower under SER than under both RAN and STR .

Figure 8 displays the result of simulations when the distribution of C is highly polarized (a mixture of $Beta(1, 5)$ and $Beta(5, 1)$) In External Appendix ?? we present additional

²⁹Specifically, $\mathbb{T}_{RAN}(x; med[C]) = \mathbb{P}(Bi[j, 0.5] \geq x)$ and $\mathbb{T}_{STR}(x; med[C]) = \mathbb{P}(Bi[j + d + p, 0.5] \geq x + p)$.

Figure 8: Selection of jurors below the median



Note: Fraction of juries with a at least given number of jurors below the median of C under STR (green dashed line) and $S\mathcal{E}R$ (continuous lines) relative to the same fraction under RAN (i.e. $\mathbb{T}_M(x; med[C]) - \mathbb{T}_{RAN}(x; med[C])$). Throughout, we fix $j = 12$, $d = p = 6$ and $C \sim r * Beta(1, 5) + (1 - r) * Beta(5, 1)$ (for $r \in \{0.1, 0.25, 0.5\}$) whereas the number of jurors below the median is on the horizontal axis. For each set of parameters, results for $S\mathcal{E}R$ are averages across 50,000 simulated jury selections, whereas values for RAN and STR are computed analytically and are independent of r (see Footnote 29).

simulations for less polarized distributions. These additional simulations suggest that high levels of polarization are required for $S\mathcal{E}R$ to more often select a majority of jurors below the median than STR . Also, for lower levels of polarization, $S\mathcal{E}R$ tends to selects fewer juries made of a majority of jurors below the median than RAN .³⁰

7.2 Representation of balanced groups

Concerns about the effect of jury selection on group-representation often focus on the representation of racial minorities. Even though the U.S. Supreme Court initially banned challenges based on race only (*Batson v. Kentucky*, 1986), it later banned challenges based on *gender* (*J.E.B. v. Alabama*, 1994). It is therefore natural to ask whether the advantage of $S\mathcal{E}R$ in terms of minority representation comes at the cost of a worse representation of gender groups.

Unlike minorities which correspond to groups of unequal sizes represented by small values of r , gender-groups can be thought of as even-sized groups and are better modeled

³⁰Because the parties' actions under $S\mathcal{E}R$ are influenced by the mean of the distribution but not in any clear way by the median (and because of the complexity of the game tree), we were unable to formalize the effect of polarization on these comparisons in terms of the model parameters.

using $r \approx 0.5$. With groups of similar sizes, both procedures almost always select at least a few members from either group. It is therefore more interesting to compare procedures directly in terms of the *proportion* of group- a jurors they select (rather than in terms of the probability of selecting *at least* x members from group- a , as we did before).

In this last section, we let $r = 0.5$ and study the expected proportion of group- a jurors selected under STR and SER . We denote these proportions r_{STR} and r_{SER} and focus on how close r_{STR} and r_{SER} are from the 50% of group- a jurors that prevail in the population.

As in the last two sections, it is not possible to generally compare STR and SER in terms of the procedures' ability to select an even proportion of group- a and group- b jurors. In some cases, r_{STR} can be further away from 50% than r_{SER} , and the converse may be true in other cases. For example, with $d = p = 6$ and $j = 12$, if $C_a \sim U[0, 1]$ and $C_b \sim Beta(1, 5)$, simulations reveal that $r_{STR} = 43.7\%$ whereas $r_{SER} = 45.8\%$. In contrast, when $C_a \sim Beta[4, 2]$ and $C_b \sim Beta(1, 5)$, $r_{STR} = 50.3\%$ whereas $r_{SER} = 52.2\%$.

These examples however suggest that, as the group distributions become more symmetrical, r_{STR} get closer to 50%. Proposition 6 confirms this pattern. If the group-distributions are symmetrical (or if they do not overlap) and if $d = p$, then $r_{STR} = 50\%$ whereas SER does not necessarily select an even proportion of jurors from each group. This is because even when $r = 50\%$ and distributions are symmetrical, the multiplicative utility function that the parties use to assess the value of a jury (a consequence of the assumption that convictions require unanimity) creates asymmetries in the use of challenges under SER .³¹

We say that random variables C_a and C_b are **symmetric** if $f_a(c) = f_b(1 - c), \forall c \in [0, 1]$.

Proposition 6. *Suppose that $r = 0.5$ and $d = p$. If (a) the two group distributions do not overlap,³² or (b) C_a and C_b are symmetric, then $r_{STR} = r_{RAN}$.*

Table 2(a) illustrates Proposition 6 and the fact that a similar statement does not hold for SER . Unlike STR , SER can select unequal numbers of group- a and group- b jurors even when distributions are symmetrical across groups. Therefore, as a consequence of Proposition 6, r_{SER} can in these cases be further away than r_{STR} from the 50% of group- a

³¹Flanagan (2015) shows that, in this symmetrical case, the asymmetry of the payoffs still forces the defendant to be more conservative than the plaintiff when using its challenges, hence leading to an uneven selection of jurors from the two groups.

³²That is either $\mathbb{P}(C_a > C_b) = 0$ or $\mathbb{P}(C_b > C_a) = 0$. The same result would apply if the two distributions did not overlap *in the limit as in Proposition 3*.

Table 2: Representation of Group-a jurors with balanced group sizes

Polarization Procedure	Extreme		Moderate		Mild		(All)
	SER	STR	SER	STR	SER	STR	RAN
Average fraction of group a	0.48	0.50	0.49	0.50	0.50	0.50	0.50
Standard deviation	0.18	0.20	0.16	0.17	0.15	0.15	0.14

(a) Group-a proportion $r = 0.5$, group distributions as in Figure 3.

Polarization Procedure	Extreme		Moderate		Mild		(All)
	SER	STR	SER	STR	SER	STR	RAN
Average fraction of group a	0.39	0.40	0.42	0.42	0.45	0.44	0.45
Standard deviation	0.18	0.20	0.16	0.17	0.15	0.15	0.14

(b) Group-a proportion $r = 0.45$, group distributions as in Figure 3.

Polarization Procedure	Extreme*		Moderate*		Mild*		(All)
	SER	STR	SER	STR	SER	STR	RAN
Average fraction of group a	0.47	0.50	0.49	0.48	0.49	0.48	0.50
Standard deviation	0.18	0.20	0.15	0.16	0.15	0.16	0.14

(c) Group-a proportion $r = 0.5$, group distributions slightly asymmetric*

In panel (c) Extreme corresponds to $C_a \sim \text{Beta}(1, 5)$ and $C_b \sim \text{Beta}(5, 2)$, Moderate* to $C_a \sim \text{Beta}(2, 4)$ and $C_b \sim \text{Beta}(4, 3)$, and Mild* to $C_a \sim \text{Beta}(3, 4)$ and $C_b \sim \text{Beta}(4, 4)$.

Note: The rows report the average number and standard deviation of group- a jury members out of 50,000 simulations of jury selection with parameters $j = 12$ and $d = p = 6$.

jurors that prevail in the population.

Table 2(a) however suggests that these differences may be quantitatively small, and that sizable differences may require high levels of polarization between groups. Table 2(b) and 2(c) also report the results of simulations in which the symmetries required for Proposition 6 to hold are slightly relaxed. These indicate that the advantage of STR in the representation of balanced groups established in Proposition 6 (i.e., the fact that r_{STR} is closer to 50% than r_{SER}) may not be robust to even mild relaxations of these symmetries. In particular, when $r = 0.45$ (Table 2(b)) or when $r = 0.5$ but the group-distributions are slightly asymmetric (Table 2(c)), r_{STR} is closer than r_{SER} to the proportion of group- a jurors that prevail in the population for some levels of polarization.

8 Conclusion

In this paper, we study the relative performance of two stylized jury-selection procedures. Strike and Replace presents potential jurors one-by-one to the parties, whereas the Struck procedure presents all potential jurors before they exercise vetoes. When jurors differ in their probability of voting for the defendant’s conviction, and on group membership, we show that when groups have polarized views Strike is more effective at excluding jurors with extreme views, but generally selects fewer members of a minority group than Strike and Replace, leading to a conflict between these two goals.

Besides the selection of juries, this research may be suggestive of applications to other contexts where the mechanisms or procedures used to select (groups of) agents may have disparate outcomes on group-representation. One example is the voting rules that hiring committees use to select job candidates for interviews and fly-outs.

Sociologists [Small and Pager \(2020\)](#) argue that systemic factors may lead to disparate outcomes even in the absence of taste-based or statistical discrimination, the traditional explanations for group inequalities in Economics. In the context modeled in this paper, it may appear natural that asymmetric group preferences generate asymmetric outcomes. Our results emphasize that the the choice of the selection procedure may exacerbate such asymmetries. This paper formalizes an example in which the pursuit of one objective, preventing extreme jurors to serve on juries, may lead to larger group disparities even if mechanisms and institutions are formally race-neutral.

A Appendix: Proofs

A.1 Preliminary technical results

Limit of a ratio of binomial probabilities

Lemma 1. *For all $\eta \in \mathbb{N}$ and any $k \in \{1, \dots, \eta - 1\}$,*

$$\lim_{\pi \rightarrow 0} \frac{\mathbb{P}[Bi(\eta, \pi) = k]}{\mathbb{P}[Bi(\eta, \pi) > k]} = \infty.$$

Proof. Using the standard formula for the p.d.f. of a binomial and the representation of the

c.d.f. of the binomial with *regularized incomplete beta function*, we can re-write the ratio as

$$\frac{\mathbb{P}[Bi(\eta, \pi) = k]}{1 - \mathbb{P}[Bi(\eta, \pi) \leq k]} = \frac{\binom{\eta}{k} \pi^k (1 - \pi)^{\eta - k}}{1 - (\eta - k) \binom{\eta}{k} \int_0^{1 - \pi} x^{\eta - k - 1} (1 - x)^k dx} \quad (1)$$

As $\pi \rightarrow 0$, both the numerator and the denominator tend to 0. We use L'Hopital's rule to complete the proof:

$$\begin{aligned} & \frac{(\partial/\partial\pi) \binom{\eta}{k} \pi^k (1 - \pi)^{\eta - k}}{(\partial/\partial\pi) \left(1 - \left[(\eta - k) \binom{\eta}{k} \int_0^{1 - \pi} x^{\eta - k - 1} (1 - x)^k dx \right] \right)} \\ &= \frac{\binom{\eta}{k} * [k\pi^{k-1}(1 - \pi)^{\eta - k} + \pi^k(\eta - k)(1 - \pi)^{\eta - k - 1}]}{- (\eta - k) \binom{\eta}{k} [(-1) * (1 - \pi)^{\eta - k - 1} \pi^k]} \\ &= \frac{k\pi^{k-1}(1 - \pi)^{\eta - k}}{(\eta - k)(1 - \pi)^{\eta - k - 1} \pi^k} + \frac{\pi^k(\eta - k)(1 - \pi)^{\eta - k - 1}}{(\eta - k)(1 - \pi)^{\eta - k - 1} \pi^k} \\ &= \frac{k(1 - \pi)}{(\eta - k)\pi} + 1 \xrightarrow{\pi \rightarrow 0} \infty \end{aligned}$$

□

Continuity of challenge thresholds in $S\mathcal{E}R$ as C^i converges in distribution

Lemma 2. Consider a sequence of random variables $\{C^i\}_{i=1}^{\infty}$ that converges in distribution to some random variable C^* . Let $t_I(\gamma, C^i)$ denote the challenge threshold used by party $I \in \{D, P\}$ in an arbitrary subgame γ of $S\mathcal{E}R$ when the distribution of conviction probabilities is C^i . For any such subgame γ , we have $\lim_{i \rightarrow \infty} t_I(\gamma, C^i) = t_I(\gamma, C^*)$.

Proof. In any subgame $\tilde{\gamma}$, $t_I(\tilde{\gamma}, C^i)$ is the ratio of the value of continuation subgames if I challenges the presented juror, or if both parties abstain from challenging (Brams and Davis, 1978). Therefore, $\lim_{i \rightarrow \infty} t_I(\gamma, C^i) = t_I(\gamma, C^*)$ follows directly if we show that the value of any subgame, which we denote $V(\gamma, C^i)$, converges to $V(\gamma, C^*)$ as i tends to infinity.³³

The latter follows directly from the recursive characterization of $V(\gamma, C^i)$ in Brams and Davis (1978). Recall that each subgame γ can be characterized by the number of jurors κ that remain to be selected, the number of challenges left to the defendant δ , and the

³³Because we assume that all distributions of conviction probabilities are continuous, there are no issues related to the possibility for the bottom of one of these ratios to converge to zero.

number of challenges left to the plaintiff π . With this notation, the recursive proof that for all $\kappa, \delta, \pi \geq 0$, $V([\kappa, \delta, \pi], C^i)$ converges to $V([\kappa, \delta, \pi], C^*)$ as i tends to infinity can be decomposed in a number of cases. Let $F^i(c)$ denote the c.d.f. of C^i , $F^*(c)$ the c.d.f. of C^* , and $F(c)$ the c.d.f. of an arbitrary distribution C , with μ^i , μ^* , and μ being the corresponding expected values. In each step, the initial formula for $V([\kappa, \delta, \pi], C^i)$ is taken from [Brams and Davis \(1978\)](#).

Case 1: $\kappa = 0, \delta \geq 0, \pi \geq 0$. In this case, $V([0, \delta, \pi], C) = 1$ for all C and the convergence of $V([0, \delta, \pi], C^i)$ to $V([0, \delta, \pi], C^*)$ follows trivially.

Case 2: $\kappa > 0, \delta = 0, \pi = 0$. In this case, $V([\kappa, 0, 0], C) = \mu^\kappa$ for all C and the convergence of $V([\kappa, 0, 0], C^i)$ to $V([\kappa, 0, 0], C^*)$ follows from the fact that C^i converges in distribution to C^* .

Case 3: $\kappa > 0, \delta = 0, \pi > 0$. In this case, for all C ,

$$V([\kappa, 0, \pi], C) = V(\kappa - 1, 0, \pi) * \left[1 - \int_{t_I([\kappa, 0, \pi], C)}^1 F(c) dc \right],$$

and $t_I([\kappa, 0, \pi], C) = V([\kappa, 0, \pi - 1], C) / V([\kappa - 1, 0, \pi], C)$. The convergence of $V([\kappa, 0, \pi], C^i)$ to $V([\kappa, 0, \pi], C^*)$ then follows recursively from the previous cases and from C^i converging in distribution to C^* .

Case 4: $\kappa > 0, \delta > 0, \pi = 0$. In this case, for all C ,

$$V([\kappa, \delta, 0], C) = V([\kappa, \delta - 1, 0], C) - V([\kappa - 1, \delta, 0], C) * \int_0^{t_D([\kappa, \delta, 0], C)} F(c) dc,$$

where $t_D([\kappa, \delta, 0], C) = V([\kappa, \delta - 1, 0], C) / V([\kappa - 1, \delta, 0], C)$. The convergence of $V([\kappa, \delta, 0], C^i)$ to $V([\kappa, \delta, 0], C^*)$ then follows recursively from the previous cases and from C^i converging in distribution to C^* .

Case 5: $\kappa > 0, \delta > 0, \pi > 0$. In this case, for all C ,

$$V([\kappa, \delta, \pi], C) = V([\kappa, \delta - 1, \pi], C) - V([\kappa - 1, \delta, \pi], C) * \int_{t_I([\kappa, \delta, \pi], C)}^{t_D([\kappa, \delta, \pi], C)} F(c) dc,$$

where $t_D([\kappa, \delta, \pi], C) = V([\kappa, \delta - 1, \pi], C) / V([\kappa - 1, \delta, \pi], C)$ and $t_I([\kappa, \delta, \pi], C) = V([\kappa, \delta, \pi - 1], C) / V([\kappa - 1, \delta, \pi], C)$. The convergence of $V([\kappa, \delta, 0], C^i)$ to $V([\kappa, \delta, 0], C^*)$

follows recursively from the previous cases and from C^i converging in distribution to C^* . ■

Comparative statics of probabilities from a symmetric binomial

Lemma 3. $\mathbb{P}[Bi(\eta + 2, 0.5) \geq k + 1] > \mathbb{P}[Bi(\eta, 0.5) \geq k]$ if and only if $k > \frac{\eta}{2} + \frac{1}{2}$.

Proof. We can decompose $\mathbb{P}[Bi(\eta + 2, 0.5) \geq k + 1]$ in terms of $Bi(\eta, 0.5)$ and $Bi(2, 0.5)$:

$$\begin{aligned}
& \mathbb{P}[Bi(\eta + 2, 0.5) \geq k + 1] \\
= & \mathbb{P}[Bi(\eta, 0.5) \geq k + 1] + \mathbb{P}[Bi(\eta, 0.5) = k] * \mathbb{P}[Bi(2, 0.5) \geq 1] + \\
& \mathbb{P}[Bi(\eta, 0.5) = k - 1] * \mathbb{P}[Bi(2, 0.5) = 2] \\
= & \mathbb{P}[Bi(\eta, 0.5) \geq k + 1] + \mathbb{P}[Bi(\eta, 0.5) = k] * 0.75 + \mathbb{P}[Bi(\eta, 0.5) = k - 1] * 0.25
\end{aligned}$$

Also,

$$\mathbb{P}[Bi(\eta, 0.5) \geq k] = \mathbb{P}[Bi(\eta, 0.5) \geq k + 1] + \mathbb{P}[Bi(\eta, 0.5) = k].$$

The last two equalities imply that $\mathbb{P}[Bi(\eta + 2, 0.5) \geq k + 1] > \mathbb{P}[Bi(\eta, 0.5) \geq k]$ iff

$$\begin{aligned}
\mathbb{P}[Bi(\eta, 0.5) = k] * 0.75 + \mathbb{P}[Bi(\eta, 0.5) = k - 1] * 0.25 &> \mathbb{P}[Bi(\eta, 0.5) = k] \\
\mathbb{P}[Bi(\eta, 0.5) = k - 1] * 0.25 &> \mathbb{P}[Bi(\eta, 0.5) = k] * 0.25 \\
\mathbb{P}[Bi(\eta, 0.5) = k - 1] &> \mathbb{P}[Bi(\eta, 0.5) = k] \\
\binom{\eta}{k-1} 0.5^{k-1} 0.5^{\eta-(k-1)} &> \binom{\eta}{k} 0.5^k 0.5^{\eta-k} \\
\frac{\eta!}{(\eta - [k-1])!(k-1)!} &> \frac{\eta!}{(\eta - k)!k!} \\
\frac{(\eta - k)!}{(\eta - [k-1])!} &> \frac{(k-1)!}{k!} \\
\frac{1}{\eta - k + 1} &> \frac{1}{k} \\
k &> \frac{\eta}{2} + \frac{1}{2}
\end{aligned}$$

■

Relationship between order statistics of symmetric distributions

For any number of draws w and any $k \leq w$, let $C_g^{k,w}$ denote the k -th order statistic out of w draws from distribution C_g , and $f_g^{k,w}(x)$ the corresponding probability density function.

Lemma 4. *Suppose that C_a and C_b are symmetric. Then, for any $w \in \mathbb{N}$ and any $k \in \{1, \dots, w\}$, we have $f_a^{k,w}(c) = f_b^{w-k+1,w}(1-c)$ for all $c \in [0, 1]$.*

Proof. Recall that, by definition, C_a and C_b being symmetric implies $f_a(c) = f_b(1-c)$ for all $c \in [0, 1]$, which, in turn, implies $F_a(c) = F_b(1-c)$ for all $c \in [0, 1]$. We therefore have,

$$\begin{aligned}
f_a^k(c) &= k \binom{w}{k} f_a(c) [F_a(c)]^{k-1} [1 - F_a(c)]^{w-k} \\
&= k \binom{w}{k} f_b(1-c) [1 - F_b(1-c)]^{k-1} [1 - (1 - F_b(1-c))]^{w-k} \\
&= k \frac{w!}{(w-k)!k!} f_b(1-c) [1 - F_b(1-c)]^{k-1} [f_b(1-c)]^{w-k} \\
&= (w-k+1) \frac{w!}{(w-k+1)!(k-1)!} f_b(1-c) [(1 - F_b(1-c))]^{k-1} [F_b(1-c)]^{w-k} \\
&= (w-k+1) \frac{w!}{(w-k+1)!(w-(w-k+1))!} f_b(1-c) [1 - F_b(1-c)]^{k-1} [F_b(1-c)]^{w-k} \\
&= (w-k+1) \binom{w}{w-k+1} f_b(1-c) [1 - F_b(1-c)]^{k-1} [F_b(1-c)]^{w-k} \\
&= f_b^{w-k+1}(1-c)
\end{aligned}$$

■

A.2 Proof of Proposition 1

Consider an arbitrary $c \in (0, 1)$ and let us refer to jurors with conviction probability no larger than c as *extreme jurors*. Let $\mathbb{T}_M(x; c|k)$ denote the probability that at least x extreme jurors are selected by procedure M conditional on there being exactly k of extreme jurors in the panel of n . By the Law of Total Probability,

$$\mathbb{T}_M(x; c) = \sum_{k=x}^n \mathbb{P} \left[Bi(n, F(c)) = k \right] \mathbb{T}_M(x; c|k). \quad (2)$$

Consider first the *STR* procedure. Note that for all c , we have $\mathbb{T}_{STR}(x; c|x) = 0$ because if there are exactly x extreme jurors in the panel, one of them is necessarily challenged by the plaintiff under *STR* (recall that $p \geq 1$). Therefore, by (2), we have

$$\mathbb{T}_{STR}(x; c) = \sum_{k=x+1}^n \mathbb{P} \left[Bi(n, F(c)) = k \right] \mathbb{T}_{STR}(x; c|k) \leq \mathbb{P} \left[Bi(n, F(c)) > x \right], \quad (3)$$

where the last inequality follows from the fact that $\mathbb{T}_{STR}(x; c|k) \in [0, 1]$ for all k (as $\mathbb{T}_{STR}(x; c|k)$ is a probability).

Next, consider procedure $S^{\mathcal{E}R}$. Our goal is to construct a lower bound for the probability of selecting an extreme juror and show that, as $c \rightarrow 0$, this lower bound does not converge to 0 as fast as (3). To do so, we introduce an decreasing function $\sigma(c) > 0$ such that, when c is sufficiently small, $\mathbb{T}_{S^{\mathcal{E}R}}(x; c|k) \geq \sigma(c)$ for any $k \geq x$. To construct σ , consider the restricted sample space in which there are k extreme jurors in the panel.

Let \underline{t}_P be the lowest challenge threshold used by the plaintiff in any subgame of $S^{\mathcal{E}R}$. Clearly, $\underline{t}_P > 0$.³⁴ Henceforth, we focus on $c \in (0, \underline{t}_P)$. We first consider the function $\alpha(c)$ defined as the probability that $c_j \in (c, \underline{t}_P)$ for *all* the $(n - k)$ non-extreme jurors in the panel. Because C is continuous and 0 is the lower-bound of its support, there exists $y > 0$ sufficiently small such that $\alpha(c) > 0$ for all $c \in [0, y]$.³⁵ Also, $\alpha(c)$ is weakly decreasing in c . By construction of \underline{t}_P , for such panels (with k extreme jurors and $c_j \in (c, \underline{t}_P)$ for all the $(n - k)$ non-extreme jurors), the plaintiff uses all its challenges on the p first jurors it is presented with, and the defendant never uses any challenges.³⁶ Hence, for these panels, the probability that all k extreme jurors are selected is the probability that none of these jurors are among the p first presented jurors, i.e., $\binom{n-p}{k} / \binom{n}{k}$. Overall, for $c \in (0, \underline{t}_P)$, we have $\mathbb{T}_{S^{\mathcal{E}R}}(x; c|k) \geq \alpha(c) * \binom{n-p}{k} / \binom{n}{k}$, and $\sigma(c) := \alpha(c) * \binom{n-p}{k} / \binom{n}{k}$ has the desired property.

Applying $\mathbb{T}_{S^{\mathcal{E}R}}(x; c|k) \geq \sigma(c)$ to (2) with $M = S^{\mathcal{E}R}$, we obtain for all c sufficiently small (specifically $c \in (0, \underline{t}_P)$)

$$\mathbb{T}_{S^{\mathcal{E}R}}(x; c) \geq \sum_{k=x}^n \mathbb{P}[Bi(n, F(c)) = k] * \sigma(c) \geq \mathbb{P}[Bi(n, F(c)) = x] * \sigma(c). \quad (4)$$

Overall, combining (3) and (4) yields

$$\lim_{c \rightarrow 0} \frac{\mathbb{T}_{S^{\mathcal{E}R}}(x; c)}{\mathbb{T}_{STR}(x; c)} \geq \lim_{c \rightarrow 0} \frac{\mathbb{P}[Bi(n, F(c)) = x] * \sigma(c)}{\mathbb{P}[Bi(n, F(c)) > x]} = \infty, \quad (5)$$

³⁴Formally, if Γ denotes the set of subgames of $S^{\mathcal{E}R}$ and $t_P(\gamma)$ the plaintiff's challenge threshold in any subgame $\gamma \in \Gamma$, then $\underline{t}_P = \min_{\gamma \in \Gamma} t_P(\gamma)$ (the minimum is well-defined since Γ is of finite size). In any subgame γ of $S^{\mathcal{E}R}$, there is always a $c > 0$ low enough such that if the juror who is presented to the parties in the first round of γ is of type c , the plaintiff will challenge that juror. Therefore, $\underline{t}_P > 0$.

³⁵Because 0 is the lower-bound of the defined support, $\mathbb{P}(C \in [0, \epsilon]) > 0$ for all $\epsilon > 0$. By continuity of C , there must therefore exist some $\delta > 0$ such that $\mathbb{P}(C \in [\delta/2, \delta]) > 0$. We then have $\alpha(c) > 0$ for all $c < \delta$.

³⁶The latter follows because in any subgame the defendant's threshold is always higher plaintiff's (in equilibrium, the defendant and the plaintiff never both want to challenge the presented juror).

where the last equality follows from Lemma 1 and the fact that $\sigma(c) > 0$ is decreasing in c .³⁷ In turn, $\lim_{c \rightarrow 0} \mathbb{T}_{S\mathcal{E}R}(x; c) / \mathbb{T}_{STR}(x; c) = \infty$ and the fact that $\lim_{c \rightarrow 0} \mathbb{T}_{S\mathcal{E}R}(x; c) = \lim_{c \rightarrow 0} \mathbb{T}_{STR}(x; c) = 0$ together imply that there exists some $\underline{c} > 0$ small enough such that $\mathbb{T}_{STR}(x; c) < \mathbb{T}_{S\mathcal{E}R}(x; c)$ for all $c \in (0, \underline{c})$.

A.3 Proof of Proposition 2

Using the same notation as in the proof of Proposition 1, we have

$$\mathbb{T}_{RAN}(x; c) \geq \mathbb{P}[Bi(n, F(c)) = x] * \mathbb{T}_{RAN}(x; c|x). \quad (6)$$

Note that $\mathbb{T}_{RAN}(x; c|x)$ is the probability that an Hypergeometric random variable with x success, $n - x$ failures, and j draws, results in the draw of exactly x successes. Therefore, $\mathbb{T}_{RAN}(x; c|x) > 0$. Finally, combining (6) and (3) yields

$$\lim_{c \rightarrow 0} \frac{\mathbb{T}_{RAN}(x; c)}{\mathbb{T}_{STR}(x; c)} \geq \lim_{c \rightarrow 0} \frac{\mathbb{P}[Bi(n, F(c)) = x] * \mathbb{T}_{RAN}(x; c|x)}{\mathbb{P}[Bi(n, F(c)) > x]} = \infty,$$

where the last equality follows from Lemma 1 and the fact that $\mathbb{T}_{RAN}(x; c|x) > 0$. The result then follows as in the proof of Proposition 1.

A.4 Proof of Proposition 3

The structure of the proof is similar to that of the previous propositions. We focus on the case we analyzed in the main paper, where the minority uniformly favors the defendant, i.e., $\lim_{i \rightarrow \infty} \mathbb{P}(C_a^i > C_b^i) = 0$. The proof for the other case is symmetrical.

As in the previous proofs, for any arbitrary triple (C_a^i, C_b^i, r^i) , we first decompose $\mathbb{A}_{STR}^i(x)$ and $\mathbb{A}_{S\mathcal{E}R}^i(x)$ by conditioning on the number of minority jurors in the panel.

First, consider STR and let us decompose $\mathbb{A}_{STR}^i(x)$ conditional, on the one hand, on the panel containing more than x minority jurors — which occurs with probability $\mathbb{P}[Bi(n, r^i) > x]$, and on the other, on the panel containing exactly x minority jurors — which occurs with probability $\mathbb{P}[Bi(n, r^i) = x]$. In the first case (i.e., more than x minority jurors in the panel), the probability that the panel contains at least x minority jurors is an upper bound

³⁷To apply Lemma 1, note that because C is continuous and the lower-bound of the support of C is 0, we have $F(c) > 0$ for all $c > 0$ and $\lim_{c \rightarrow 0} F(c) = 0$.

on the probability that STR selects them. In the second case (i.e., exactly x minority jurors in the panel), STR selects at least x minority jurors provided that none of the minority jurors in the panel are challenged. This occurs with a probability no larger than the probability that the lowest conviction-probability among minorities is larger than the p -th conviction probability among majority jurors (since the latter is required for the plaintiff not to challenge any of the minority jurors in the panel). Recall that for any number of draws w and any $k \leq w$, we let $C_g^{k,w}$ denote the k -th order statistic out of w draws from group $g \in \{a, b\}$. With this notation, we therefore have,

$$\mathbb{A}_{STR}^i(x) \leq \mathbb{P}[Bi(n, r^i) > x] + \mathbb{P}[Bi(n, r^i) = x] * \mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x}). \quad (7)$$

Note that because $\lim_{i \rightarrow \infty} \mathbb{P}(C_a^i > C_b^i) = 0$, we have $\lim_{i \rightarrow \infty} \mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x}) = 0$.

Second, consider SER . Clearly, $\mathbb{A}_{SER}^i(x)$ is no smaller than the probability for SER to select at least x minority jurors when there are exactly x minority jurors in the panel. The latter is equal to $\mathbb{P}[Bi(n, r^i) = x] * \sigma(x; r^i, C_a^i, C_b^i)$, where $\sigma(x; r^i, C_a^i, C_b^i)$ denotes the probability that SER selects x minority jurors *conditional* on having x minority jurors in the panel, as a function of r^i , C_a^i , and C_b^i . In summary, with this notation, we have,

$$\mathbb{A}_{SER}^i(x) \geq \mathbb{P}[Bi(n, r^i) = x] * \sigma(x; r^i, C_a^i, C_b^i). \quad (8)$$

We now show that $\lim_{i \rightarrow \infty} \sigma(x; r^i, C_a^i, C_b^i) > 0$. For all $i \in \mathbb{N}$, let $C^i = r^i C_a^i + (1 - r^i) C_b^i$. Observe that because $\lim_{i \rightarrow \infty} r_i = 0$ and because C_b^i converges in distribution to C_b^* , C^i converges in distribution to C_b^* . By Lemma 2, this implies that for any subgame γ of SER and both $I \in \{D, P\}$, we have $\lim_{i \rightarrow \infty} t_I(\gamma, C^i) = t_I(\gamma, C_b^*)$. Note that $t_I(\gamma, C_b^*)$ lies in the interior of the support of C_b^* for both $I \in \{D, P\}$. Also recall that in the limit, the supports of C_a^i and C_b^i do not overlap as we have $\mathbb{P}(C_a^* > C_b^*) = 0$. Therefore, in the limit, the defendant never challenges a minority juror, which in turn implies that

- (a) as i tends to infinity, the probability that the defendant challenges one of the x minority jurors in the panel tends to zero.

Because $t_I(\gamma, C_b^*)$ lies in the interior of the support of C_b^* for both $I \in \{D, P\}$, there is also a range of conviction probabilities $[\underline{c}, \bar{c}]$ low enough inside the support of C_b^* such that $P(C_b^* \in [\underline{c}, \bar{c}]) > 0$ and P challenged the juror presented in subgame γ if her conviction

probability lies within $[\underline{c}, \bar{c}]$. Furthermore, the probability that a juror with $c \in [\underline{c}, \bar{c}]$ is a majority juror is strictly positive (and tends to one as $i \rightarrow \infty$). Overall, in the limit,

- (b) the probability that the plaintiff challenges a majority juror presented in subgame γ is strictly positive.

Combining (a) and (b), in the limit and given a panel containing x minority jurors, there is a positive probability that p majority jurors are presented first, are all challenged by P , and are followed by the x minority jurors which are left unchallenged by the parties (resulting in a jury composed of at least x minority jurors). That is, $\lim_{i \rightarrow \infty} \sigma(x; r^i, C_a^i, C_b^i) > 0$.

We are now equipped to complete the proof. Combining (7) and (8) yields

$$\begin{aligned}
& \lim_{i \rightarrow \infty} \frac{\mathbb{A}_{STR}^i(x)}{\mathbb{A}_{S\mathcal{E}R}^i(x)} \\
& \leq \lim_{i \rightarrow \infty} \frac{\mathbb{P}[Bi(n, r^i) > x] + \mathbb{P}[Bi(n, r^i) = x] * \mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x})}{\mathbb{P}[Bi(n, r^i) = x] * \sigma(r^i, C_a^i, C_b^i)} \\
& = \lim_{i \rightarrow \infty} \frac{\mathbb{P}[Bi(n, r^i) > x]}{\mathbb{P}[Bi(n, r^i) = x] * \sigma(r^i, C_a^i, C_b^i)} + \frac{\mathbb{P}[Bi(n, r^i) = x] * \mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x})}{\mathbb{P}[Bi(n, r^i) = x] * \sigma(r^i, C_a^i, C_b^i)} \\
& = \lim_{i \rightarrow \infty} \frac{\mathbb{P}[Bi(n, r^i) > x]}{\mathbb{P}[Bi(n, r^i) = x]} * \frac{1}{\sigma(r^i, C_a^i, C_b^i)} + \frac{\mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x})}{\sigma(r^i, C_a^i, C_b^i)} \\
& = \underbrace{\lim_{i \rightarrow \infty} \frac{\mathbb{P}[Bi(n, r^i) > x]}{\mathbb{P}[Bi(n, r^i) = x]}}_{=0, \text{ by Lemma 1}} * \underbrace{\lim_{i \rightarrow \infty} \frac{1}{\sigma(r^i, C_a^i, C_b^i)}}_{< \infty, \text{ by } \lim_{i \rightarrow \infty} \sigma(x; r^i, C_a^i, C_b^i) > 0} + \underbrace{\lim_{i \rightarrow \infty} \frac{\mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x})}{\sigma(r^i, C_a^i, C_b^i)}}_{=0, \text{ by } \lim_{i \rightarrow \infty} \mathbb{P}([C_a^i]^{1,x} > [C_b^i]^{p,n-x}) = 0, \text{ and } \lim_{i \rightarrow \infty} \sigma(x; r^i, C_a^i, C_b^i) > 0} = 0
\end{aligned}$$

In turn, $\lim_{i \rightarrow \infty} \mathbb{A}_{STR}^i(x)/\mathbb{A}_{S\mathcal{E}R}^i(x) \leq 0$ and $\lim_{i \rightarrow \infty} \mathbb{A}_{STR}^i(x) = \lim_{i \rightarrow \infty} \mathbb{A}_{S\mathcal{E}R}^i(x) = 0$ together imply that $\exists k$ sufficiently large such that $\mathbb{A}_{S\mathcal{E}R}^i(x) > \mathbb{A}_{STR}^i(x)$ for all $i > k$.

A.5 Proof of Proposition 4

The structure of the proof is similar to that of the previous propositions. Observe that (3) and (4) are true regardless of the number of challenges awarded to the parties in STR or $S\mathcal{E}R$. That is, by the same arguments as in the proof of Proposition 1, the following two

inequalities hold regardless of the values of w , y , $\mathbb{A}_{STR-w}(x)$, or $\mathbb{A}_{SEY-y}(x)$,³⁸

$$\begin{aligned}\mathbb{T}_{STR-w}(x; c) &= \sum_{k=x+1}^n \mathbb{P}[Bi(n, F(c)) = k] \mathbb{T}_{STR-w}(x; c|k) \leq \mathbb{P}[Bi(n, F(c)) > x], \\ \mathbb{T}_{SEY-y}(x; c) &\geq \sum_{k=x}^n \mathbb{P}[Bi(n, F(c)) = k] * \sigma(c) \geq \mathbb{P}[Bi(n, F(c)) = x] * \sigma(c).\end{aligned}\tag{9}$$

The proof follows as in the proof of Proposition 1 (in particular, see (5)).

A.6 Proof of Proposition 5

The probability that *STR* selects at least x jurors with conviction-probability above the median is the probability that at least $x + d$ of the jurors in the panel have conviction-probability above the median (since d of these jurors are challenged by the defendant). Because $d = p$, for any $x \in \{1, \dots, n\}$, we therefore have

$$\mathbb{T}_{STR}(x; med[C]) = P[Bi(j + d + p, 0.5) \geq x + d] = P[Bi(j + 2d, 0.5) \geq x + d]$$

In contrast, we have

$$\mathbb{T}_{RAN}(x; med[C]) = P[Bi(j, 0.5) \geq x].$$

Therefore, by repeated application of Lemma 3, $x > (n/2) + (1/2)$ implies $\mathbb{T}_{STR}(x; med[C]) > \mathbb{T}_{RAN}(x; med[C])$. Since n is integer-valued, the last inequality corresponds to $x \geq n/2 + 1$ if n is even and $x \geq n/2 + 1.5$ if n is odd.

A.7 Proof of Proposition 6

Part (a). Under *STR*, since the group-distributions do not overlap, each party first uses all of its challenges on one of the two groups before challenging the lowest conviction probability jurors from the other group. For concreteness and without loss of generality, suppose that group a favors the defendant (i.e., $\mathbb{P}(C_a > C_b) = 0$). Let m denote the number of jurors from group- a in the panel.

Note that because $r = 0.5$, the probability that $m = k$ is the same as the probability that $m = n - k$ for all $k \in \{1, \dots, \lfloor n/2 \rfloor\}$. Also, because $d = p$, the number of group- a jurors

³⁸Recall that the proposition assumes $w, y \geq 1$.

who are selected when $m = k$ is equal to the number of group- b jurors who are selected when $m = n - k$.³⁹ Therefore, the expected number of group- a jurors in the jury selected by STR is exactly $j/2$.

Part (b). The proof is similar to the proof of Part (a). Consider the set of panel configurations $\{a, b\}^n$ where, for example, vector $(a, b, a, \dots, b, b, b) \in \{a, b\}^n$ indicates that the juror with the lowest conviction probability in the panel is a group- a juror, the juror with second-lowest conviction probability is a group- b juror, the juror with the third-lowest conviction probability is a group- a juror, ..., and the jurors with the three highest conviction probabilities are all group- b jurors. To explain the structure of the proof, suppose that n is even (we explain below how the argument generalizes to any n). We first construct a partition of $\{a, b\}^n$ into two subsets S^a and S^b of equal size and construct a bijection q between S^a and S^b . We then show that for every panel configuration $l \in S^a$ which results in m^l group- a jurors being selected, (a) the panel configuration $q[l]$ result $j - m^l$ group- a jurors being selected, and (b) panel configurations l and $q[l]$ are equally likely. As in the proof of Part (b), the result then follows directly.

Similar to the proof of Part (b), the bijection $q[l]$ is obtained by (i) mirroring l around the $\lfloor n/2 \rfloor$ position, and (ii) inverting the group of each juror in the resulting panel configuration. For example, panel configuration $q[(a, a, b, a)]$ is obtained by mirroring (a, a, b, a) around position $\lfloor n/2 \rfloor$, which results in (a, b, a, a) , and then inverting the group of each jurors in (a, b, a, a) , which results in (b, a, b, b) . Formally, if $inv[l]$ denotes the configuration that results from turning all the a 's in l into b 's and all the b 's in l into a 's, then $q[(l_1, l_2, \dots, l_{n-1}, l_n)] = inv[(l_n, l_{n-1}, \dots, l_2, l_1)]$.

Let S^a and S^b be two sets that together contain all l for which $l \neq q[l]$ and are such that $l \in S^i$ implies $q[l] \notin S^i$. Since $q[q[l]] = l$, the sets S^a and S^b have equal sizes. Also let S^* contain all l for which $l = q[l]$, if any ($S^* \neq \emptyset$ if and only if n is even). Note that $\{S^a, S^b, S^*\}$ forms of partition of $\{a, b\}^n$. Therefore, if we let $(\#m|l)$ denote the number of group- a juror that are selected conditional on configuration l and $\mathbb{P}(l)$ the probability of

³⁹First, suppose that $k \leq p$. Then, if $m = k$, no jurors from group- a (and j jurors from group- b) are selected, whereas if $m = n - k$, no jurors from group- b (and j jurors from group- a) are selected. Second, suppose that $k \in \{p + 1, \dots, \lfloor n/2 \rfloor\}$. Then, if $m = k$, $k - p = k - d$ jurors from group- a (and $j - (k - p) = j - (k - d)$ jurors from group- a) are selected, whereas if $m = n - k$, $k - d = k - p$ jurors from group- b (and $j - (k - d) = j - (k - p)$ jurors from group- b) are selected.

configuration l , we have

$$r_{STR} = \sum_{l \in S^a} \mathbb{P}(l) * (\#m|l) + \mathbb{P}(q[l]) * (\#m|q[l]) + \sum_{l \in S^*} \mathbb{P}(l) * (\#m|l).$$

Part (b) then follows from the fact that (A) $\mathbb{P}(l) = \mathbb{P}(q[l])$ for all $l \in S^a$, (B) $(\#m|l) = n - (\#m|q[l])$ for all $l \in S^a$, and (C) $(\#m|l) = j/2$ for all $l \in S^*$.

Properties (B) and (C) follow directly from the construction of q and the fact that $d = p$. Property (A), on the other hand, follows from Lemma 4 which establishes the symmetry of order statistics for symmetric distributions. A formal proof of (A) using Lemma 4 requires heavy and tedious notation. Instead, we show how (A) follows from Lemma 4 in a simple example that clarifies how the argument generalizes to other cases.

Consider the case of (a, a, b) for which $q[(a, a, b)] = (a, b, b)$. We can obtain the probability of any configuration by integrating the p.d.f. of the appropriate order statistics from the bottom to the top of $[0, 1]$. For example, using the notation for order statistics introduced before Lemma 4, we have

$$\begin{aligned} \mathbb{P}[(a, a, b)] &= \mathbb{P}[m = 2] * P[(a, a, b)|m = 2] \\ &= \mathbb{P}[Bi(3, 0.5) = 2] * \int_a^1 f_a^{1,2}(x) \left[\int_x^1 f_a^{2,2}(y) \left(\int_y^1 f_b^{1,1}(w) dw \right) dy \right] dx. \end{aligned} \quad (10)$$

We can also obtain the probability of any configuration by reverting the list of order statistics and integrating from the top to the bottom of $[0, 1]$. For example,

$$\begin{aligned} \mathbb{P}[(a, b, b)] &= \mathbb{P}[m = 1] * P[(a, b, b)|m = 1] \\ &= \mathbb{P}[Bi(3, 0.5) = 1] * \int_a^1 f_b^{2,2}(1-x) \left[\int_x^1 f_b^{1,2}(1-y) \left(\int_y^1 f_a^{1,1}(1-w) dw \right) dy \right] dx. \end{aligned} \quad (11)$$

Finally, by Lemma 4, $f_a^{1,2}(x) = f_b^{2,2}(1-x)$, $f_a^{2,2}(y) = f_b^{1,2}(1-y)$, and $f_b^{1,1}(w) = f_a^{1,1}(1-w)$, which together with symmetry of the binomial with 0.5 probability of success implies that the expressions in (10) and (11) are equal.

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